International Journal of Science and Technology (IJST)
ISSN: 3049-1118, Volume- 3, Issue- 3 (July - September 2026)

Reactor Period–Reactivity Modelling and Cross-Validation of In-Hour and PRCT Methods for the LEU Nigeria Research Reactor-1

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Abstract

Accurate conversion of reactor period to reactivity is essential for control-rod calibration and reactor-kinetics interpretation in low-power research reactors. This study cross-validates the In-hour equation and Period Reactivity Conversion Table (PRCT) using secondary low-power calibration data from the low-enriched uranium Nigeria Research Reactor-1 (LEU NIRR-1), and develops a reproducible reactor-specific period–reactivity model. Six reactor periods (45–180 s) were analysed. In-hour estimates ranged from 5.64 to 1.28 mk, whereas PRCT estimates ranged from 5.50 to 1.20 mk. The paired methods showed close agreement (mean absolute error = 0.120 mk; RMSE = 0.122 mk; mean absolute percentage deviation = 4.38%). Linear agreement analysis gave $R^2 = 0.99994$, with a slope of 1.0140 (95% CI 1.0032–1.0247). Exploratory Bland–Altman analysis showed a positive bias of 0.120 mk and 95% limits of agreement of 0.069 to 0.171 mk. To make the predictive model fully reproducible, the arithmetic mean of each In-hour/PRCT pair was fitted to a power law, reproducing the dissertation model as $\rho = 337.48T^{-1.070}$. The fit was strong ($R^2 = 0.9963$; adjusted $R^2 = 0.9954$), and leave-one-out cross-validation yielded RMSE = 0.159 mk and MAPE = 5.00%. The model exponent was -1.070(95% CI $-1.161$to -0.980), supporting an approximately inverse period–reactivity relationship over the investigated interval. The two conventional methods are therefore mutually consistent for the analysed LEU NIRR-1 period range, while the fitted model is best regarded as a compact interpolation and cross-checking tool. It is recommended that independent validation with new reactor measurements is required before operational use.

Keywords: LEU NIRR-1; reactor period; reactivity; In-hour equation; PRCT; control rod calibration; reactor kinetics; power-law model

1. Introduction

Reactivity control is a central requirement in the safe and predictable operation of research reactors. It governs the departure of the neutron population from the critical state and therefore determines both the direction and the rate of reactor-power change. Control rods provide the principal mechanical means of adding or removing neutron absorption from the core. Their calibration supports start-up, shut-down, power manoeuvring, verification of shut-down margin, operator training and the periodic confirmation of parameters important to safety.

*Corresponding Author Email: ejoseph@fudutsinma.edu.ng
Published: 22 August 2026
DOI: https://doi.org/10.70558/IJST.2026.v3.i3.241320

International guidance consequently treats control-element calibration and reactor-period measurements as fundamental reactor-physics and commissioning activities (IAEA, 2016; IAEA, 2023).

For small positive reactivity insertions at low power, the reactor period is a convenient and sensitive observable. Once the power transient has reached an approximately stable exponential regime, the period can be related to reactivity through the point-kinetics equations. The In-hour equation supplies the standard analytical relation and explicitly incorporates the prompt-neutron generation time together with the delayed-neutron fractions and precursor decay constants (Keepin, 1965; Lamarsh & Baratta, 2001). In routine reactor work, the same relationship is often represented by a Period–Reactivity Conversion Table (PRCT), allowing a measured period to be converted to reactivity without repeatedly solving the full equation. Because tabulated values are discrete and depend on the underlying kinetics parameters, small differences may arise relative to continuous In-hour evaluations; quantifying those differences under identical conditions is therefore of practical interest.

The Nigeria Research Reactor-1 (NIRR-1) is a tank-in-pool Miniature Neutron Source Reactor (MNSR) located at the Centre for Energy Research and Training, Ahmadu Bello University, Zaria. Originally commissioned with highly enriched uranium (HEU) fuel, it was converted to low-enriched uranium (LEU) as part of international efforts to minimise civilian use of weapons-usable material. Earlier reactor-physics studies established detailed computational descriptions of the original HEU core and supported the subsequent conversion programme (Jonah et al., 2007). The International Atomic Energy Agency later documented reactor-specific analyses supporting the conversion of MNSRs, including NIRR-1, from HEU to LEU fuel (IAEA, 2018). More recent modelling of the as-built LEU core has shown close correspondence between calculated and commissioning neutronic parameters, reinforcing the value of reactor-specific validation after conversion (Asuku et al., 2023).

Control-rod worth and related reactivity parameters remain safety-relevant quantities that may be affected by changes in core configuration, irradiation history and reactor modifications, as they therefore require periodic re-evaluation. Ibrahim et al. (2022) compared experimental and analytical control-rod worth estimates for NIRR-1 and found close agreement for the investigated configuration. Complementary computational work has examined point-kinetics parameters (effective delayed-neutron fraction, prompt-neutron lifetime and mean generation time) relevant to the LEU core (Abubakar et al., 2022).

Unlike earlier NIRR-1 investigations that primarily focused on experimental control-rod calibration or computational prediction of neutronic parameters, the present study evaluates the internal agreement of two period-based reactivity-conversion methods using an identical LEU NIRR-1 dataset and extends the analysis through reactor-specific predictive modelling. The methodological contribution lies in combining conventional In-hour and PRCT calculations with quantitative method-agreement statistics, parameter uncertainty estimation, residual diagnostics, and out-of-sample cross-validation. This framework permits a clearer distinction between agreement of established reactivity-conversion methods and the predictive capability of the fitted period–reactivity model.

Accordingly, this study compares paired In-hour and PRCT estimates using mean absolute error (MAE), root-mean-square error (RMSE), percentage deviation, regression analysis, and exploratory Bland–Altman statistics. It further reconstructs a power-law model relating reactor period to reactivity from the arithmetic mean of the paired estimates, quantifies confidence intervals for the fitted coefficients, examines residual behaviour, and evaluates predictive stability using leave-one-out cross-validation. These steps strengthen the distinction between descriptive agreement and predictive validation while retaining the original dataset and reactor-physics framing. By integrating established reactor-kinetics formulations with modern method-agreement and predictive-validation procedures, the study provides a more rigorous framework for period-based reactivity determination in the LEU NIRR-1 and complements ongoing experimental and computational efforts to characterise the reactor following its conversion to LEU fuel.

2. Materials and Methods

2.1 Reactor description and experimental dataset

The Nigeria Research Reactor-1 (NIRR-1) is a tank-in-pool Miniature Neutron Source Reactor (MNSR) operated at the Centre for Energy Research and Training, Ahmadu Bello University, Zaria. Following conversion from highly enriched uranium (HEU) to low-enriched uranium (LEU) fuel, the core consists of UO_2 fuel pins clad in Zircaloy-4, with a single central cadmium control rod, annular and bottom beryllium reflectors, and light-water moderator/coolant (Figure 1). Nominal thermal power is 34 kW (Jonah et al., 2007; IAEA, 2018; Asuku et al., 2023; Ashezua et al., 2024).

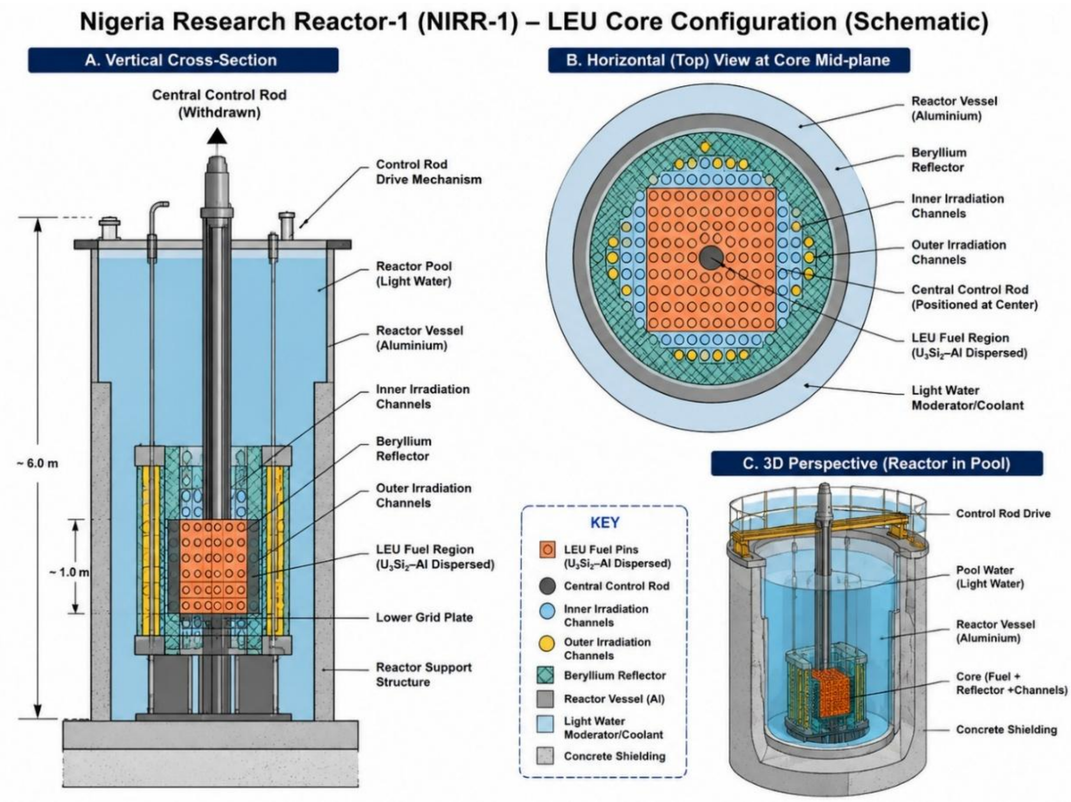

Figure 1. Schematic representation of the LEU Nigeria Research Reactor-1 (NIRR-1), showing the compact reactor core, central control rod, beryllium reflector, irradiation channels, reactor vessel, and surrounding light-water moderator/coolant.

The present study utilises a low-power positive-period dataset obtained from controlled control-rod withdrawals under near-critical, low-flux conditions. For each withdrawal, the stable reactor period T(s) was determined from measured doubling times after transient effects had decayed, yielding a set of paired period observations that form the basis for subsequent reactivity conversion.

2.2 Research design

An experimental-analytical design based on secondary reactor calibration data was adopted. Reactor periods previously obtained through the positive-period method were subjected to two independent reactivity-conversion procedures:

i. analytical determination using the In-hour equation; and

ii. tabulated determination using the Period–Reactivity Conversion Table (PRCT).

The resulting reactivity values, which quantify the departure of the reactor from criticality, were subsequently compared to determine the consistency of the two reactor-kinetics approaches. Curve fitting was then applied to establish the physical dependence of reactivity on reactor period, reflecting the inverse relationship predicted by point-reactor kinetics. Hence, the methodology integrates reactor-period measurement, neutron-kinetics-based reactivity determination, and quantitative evaluation of the period–reactivity relationship, providing both a physical and statistical basis for characterizing the reactivity behaviour of the LEU

2.3 In-hour equation

Reactivity ρ corresponding to a measured stable period T was evaluated using the six-group In-hour equation derived from the point-kinetics model (Keepin, 1965; Lamarsh & Baratta, 2001):

$$\rho = \frac{\Lambda}{T} + \sum_{i=1}^{6} \frac{\beta_i}{1+\lambda_i T} \tag{1}$$

where Λ is the prompt-neutron generation time (s), β_i are the effective delayed-neutron fractions of the six precursor groups, and $\lambda_i$are the corresponding decay constants (s^{-1}). Reactor-specific kinetics parameters appropriate to the LEU core were adopted (Abubakar et al., 2022). Reactivity is expressed in units of milli-k (mk), where 1 mk $= 10^{-3}\ \Delta k/k$.

2.4 Period–Reactivity Conversion Table (PRCT)

An independent set of reactivity values was obtained by direct look-up (or linear interpolation where necessary) from a pre-established Period–Reactivity Conversion Table constructed for the same delayed-neutron parameters and generation time used in the In-hour calculations. The PRCT therefore provides a discrete representation of the continuous In-hour relationship and serves as a practical operational tool.

2.5 Method-agreement statistics

Paired reactivity estimates $\rho_{\mathrm{IH},j}$ (In-hour) and $\rho_{\mathrm{PRCT},j}$ (PRCT) for each of the N observed periods were compared using the following metrics:

Mean absolute error

$$\mathrm{MAE} = \frac{1}{N}\sum_{j=1}^{N} \mid \rho_{\mathrm{IH},j} - \rho_{\mathrm{PRCT},j} \mid \tag{2}$$

Root-mean-square error

$$\mathrm{RMSE} = \sqrt{\frac{1}{N}\sum_{j=1}^{N}(\rho_{\mathrm{IH},j} - \rho_{\mathrm{PRCT},j})^2} \tag{3}$$

Percentage deviation for each pair

$$\delta_j = 100 \times \frac{\rho_{\mathrm{IH},j} - \rho_{\mathrm{PRCT},j}}{\rho_{\mathrm{PRCT},j}} \ (\%) \tag{4}$$

Ordinary least-squares regression of ρ_{IH} on ρ_{PRCT} was performed to quantify linear association and systematic bias. Exploratory Bland–Altman analysis was conducted by plotting the difference $\rho_{\mathrm{IH}} - \rho_{\mathrm{PRCT}}$ against the mean of the two estimates; the mean difference (bias) and 95 % limits of agreement (mean difference $\pm 1.96 \times$ standard deviation of the differences) were computed (Bland & Altman, 1986).

2.6 Power-law modelling

A reactor-specific empirical relationship between period and reactivity was constructed by taking the arithmetic mean of each paired estimate as the response variable:

$$\bar{\rho}_j = \frac{\rho_{\mathrm{IH},j} + \rho_{\mathrm{PRCT},j}}{2} \tag{5}$$

A power-law model of the form

$$\bar{\rho} = a\,T^b \tag{6}$$

was fitted by linear regression after logarithmic transformation:

$$\ln \bar{\rho} = \ln a + b \ln T \tag{7}$$

Parameter estimates $\hat{a}$ and $\hat{b}$ together with their standard errors and 95 % confidence intervals were obtained by ordinary least-squares methods. Model adequacy was assessed through residual plots, normality diagnostics and the coefficient of determination R^2.

2.7 Predictive validation

Leave-one-out cross-validation (LOOCV) was employed to evaluate out-of-sample predictive performance. For each observation $j = 1, \dots, N$, the power-law model was refitted to the remaining $N - 1$points and used to predict $\bar{\rho}_j$. Prediction error metrics (LOO-MAE and LOO-RMSE) were then calculated across all held-out points, providing an estimate of predictive stability independent of the training data.

3. Results and Discussion

3.1 Reactor Period–Reactivity Behaviour

The reactor periods and corresponding reactivity estimates obtained using the In-hour equation and Period–Reactivity Conversion Table (PRCT) are presented in Table 1. Across the six investigated reactor periods, both methods exhibited a monotonic decrease in reactivity with increasing reactor period. The In-hour reactivity decreased from 5.64 mk at 45 s to 1.28 mk at 180 s, whereas the corresponding PRCT values decreased from 5.50 to 1.20 mk. The absolute differences between the two methods remained small, ranging from 0.08 to 0.15 mk, while the percentage deviation decreased from 6.67% at 180 s to 2.55% at 45 s.

This inverse period–reactivity behaviour is consistent with point-reactor kinetics and the dominant influence of delayed neutrons for small positive reactivity insertions. An increase in positive reactivity produces a more rapid exponential increase in neutron population and reactor power, thereby resulting in a shorter stable reactor period. Conversely, smaller positive reactivity insertions produce slower neutron-population growth and longer reactor periods (Keepin, 1965; Lamarsh & Baratta, 2001). The observed decrease in reactivity from approximately 5.6 mk at 45 s to approximately 1.2 mk at 180 s therefore represents the expected kinetic response of the LEU NIRR-1 under the investigated low-power conditions.

The In-hour estimates were consistently higher than the corresponding PRCT values throughout the investigated period range. This systematic behaviour suggests a small method-dependent offset rather than random disagreement. One possible explanation is the difference between continuous analytical evaluation of the In-hour equation and the discrete tabulated or interpolated nature of the PRCT. The In-hour formulation explicitly incorporates prompt-neutron generation time, delayed-neutron fractions, and precursor decay constants, making the calculated reactivity sensitive to the adopted reactor-kinetics parameters (Keepin, 1965; Lamarsh & Baratta, 2001). Recent analysis of NIRR-1 has similarly demonstrated the importance of effective delayed-neutron fraction, prompt-neutron lifetime, and neutron generation time in characterising the reactor's kinetics following core conversion (Abubakar et al., 2022).

The decreasing percentage deviation toward shorter reactor periods in this study is also noteworthy. Although prompt-neutron contributions become relatively more important as the reactor period decreases (Keepin, 1965), the present six-point dataset is insufficient to establish whether the observed reduction in percentage deviation represents a systematic physical dependence on reactor period. Consequently, this trend is considered descriptive, and a larger calibration dataset would be required to establish its statistical and reactor-physics significance.

Table 1. Reactor period and paired reactivity estimates extracted from the dissertation, with the reproducibly refitted power-law model.

T (s)	In-hour (mk)	PRCT (mk)	Pair mean (mk)	Model (mk)	|Δ| (mk)	Deviation (%)
180	1.28	1.20	1.240	1.300	0.08	6.67
130	1.95	1.85	1.900	1.842	0.10	5.41
95	2.73	2.60	2.665	2.577	0.13	5.00
70	3.72	3.60	3.660	3.574	0.12	3.33
55	4.65	4.50	4.575	4.627	0.15	3.33
45	5.64	5.50	5.570	5.735	0.14	2.55

3.2 Agreement Between In-Hour and PRCT Reactivity Estimates

The quantitative agreement statistics are summarized in Table 2. The paired In-hour and PRCT estimates produced a mean absolute error (MAE) of 0.120 mk, root-mean-square error (RMSE) of 0.122 mk, and mean absolute percentage deviation of 4.38%. The standard deviation of the paired differences was 0.026 mk. These relatively small error measures indicate close numerical correspondence between the two reactivity-conversion procedures over the investigated reactor-period range. However, because all paired differences were positive, the deviations are not randomly distributed about zero but indicate a small systematic tendency for the In-hour method to produce higher reactivity estimates.

Table 2. Expanded agreement and model-validation statistics recalculated from the six final observations. The Bland–Altman limits and confidence intervals are exploratory because n = 6.

Metric	Value
Mean absolute error, In-hour vs PRCT	0.120 mk
RMSE, In-hour vs PRCT	0.122 mk
Mean absolute percentage deviation	4.38%
Mean paired bias (In-hour − PRCT)	0.120 mk
SD of paired differences	0.026 mk
Exploratory 95% limits of agreement	0.069 to 0.171 mk
Agreement regression slope (95% CI)	1.0140 (1.0032–1.0247)
Agreement regression intercept (95% CI)	0.0752 (0.0371–0.1133) mk
Agreement regression R^2	0.99994
Power-law coefficient a (95% CI)	337.48 (225.31–505.50)
Power-law exponent b (95% CI)	-1.0705 (-1.1608–-0.9801)
Power-law R^2 / adjusted R^2	0.9963 / 0.9954
Power-law in-sample RMSE	0.093 mk
LOOCV RMSE / MAE / MAPE	0.159 mk / 0.137 mk / 5.00%

Regression analysis further demonstrates the correspondence between the two methods. Regression of In-hour reactivity on PRCT reactivity yielded a slope of 1.0140 (95% CI: 1.0032–1.0247), an intercept of 0.0752 mk (95% CI: 0.0371–0.1133), and a coefficient of determination of $R^2 = 0.99994$. The slope is close to unity, while the intercept is small relative to the investigated reactivity range of approximately 1.2–5.6 mk. As shown in Figure 2, the fitted regression therefore closely follows the line of identity, indicating very strong linear correspondence between the two period-based methods. The slight displacement of the regression line above the identity line is consistent with the systematically higher In-hour estimates.

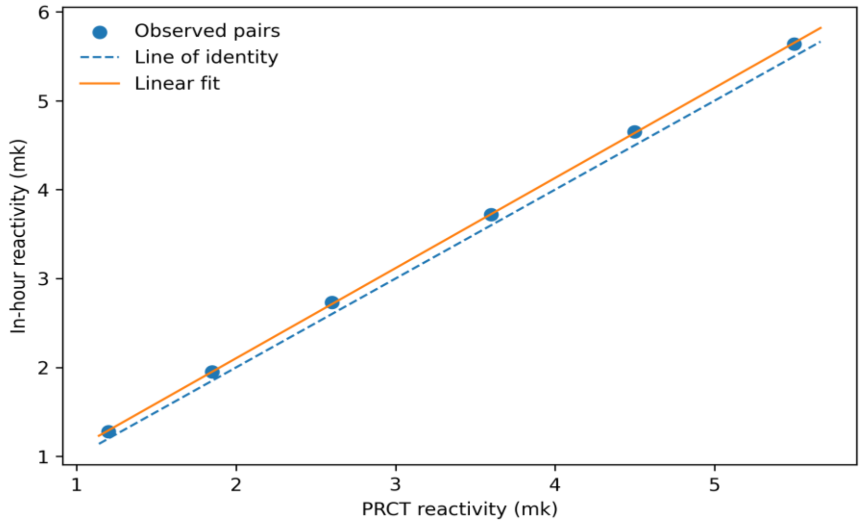

Figure 2. Method-agreement plot for In-hour versus PRCT reactivity estimates for the LEU NIRR-1. The fitted regression lies close to the line of identity (R^2 = 0.99994).

The strong linear relationship should not, however, be interpreted independently as evidence of method agreement, because correlation measures association rather than equivalence between two measurement or estimation procedures. Bland and Altman (1986) emphasized the importance of examining paired differences and their limits of agreement when evaluating agreement between methods. Accordingly, an exploratory Bland–Altman analysis was applied to the paired In-hour and PRCT estimates.

As shown in Figure 3, the mean paired difference (In-hour − PRCT) was 0.120 mk, with estimated 95% limits of agreement ranging from 0.069 to 0.171 mk. All six paired differences fell within these limits, suggesting close agreement within the investigated dataset while confirming a small positive bias of the In-hour method relative to PRCT. Because only six paired observations were available, these limits should be regarded as exploratory rather than definitive estimates of population-level agreement (Bland & Altman, 1986).

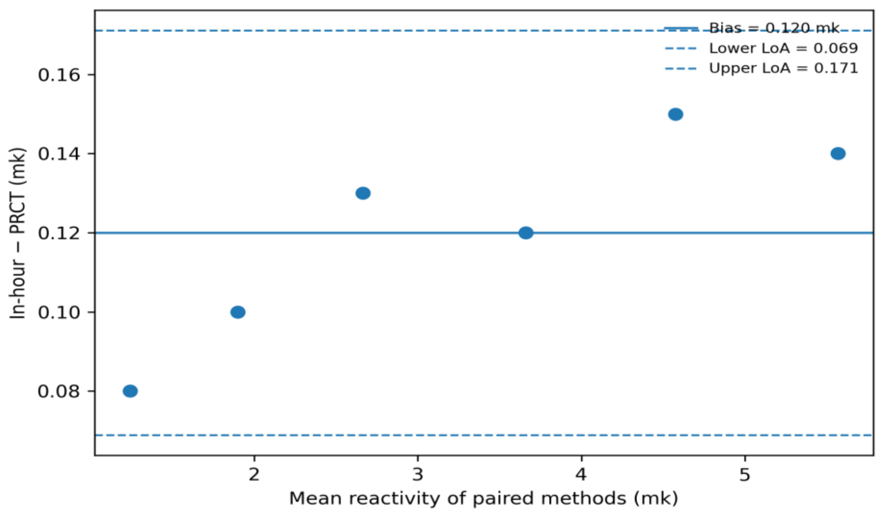

Figure 3. Bland–Altman plot showing agreement between In-hour and PRCT reactivity estimates for the LEU NIRR-1. The solid horizontal line represents the mean bias (0.120 mk), while the dashed lines indicate the lower and upper 95% limits of agreement (0.069 and 0.171 mk, respectively).

The observed agreement is consistent with the broader pattern of reactor-specific validation reported for NIRR-1. Ibrahim et al. (2022), for example, obtained close correspondence between experimental and analytical determinations of control-rod worth following reactor modification. Although that study addressed control-rod worth rather than direct In-hour–PRCT comparison, it demonstrates the value of independently cross-checking reactivity-related parameters in NIRR-1. Similarly, reactor-specific validation has become increasingly important following conversion of NIRR-1 from HEU to LEU fuel (IAEA, 2018; Asuku et al., 2023).

3.3 Power-Law Modelling and Physical Interpretation

Fitting the arithmetic mean of the paired In-hour and PRCT reactivity estimates to the power-law relationship defined earlier yielded $a = 33.48$ and $b = -1.0705$, giving

$$\rho = 337.48T^{-1.0705}$$

The model explained 99.63% of the variance in the log-transformed pair-mean reactivity ($R^2 = 0.9963$; adjusted $R^2 = 0.9954$). The 95% confidence interval for the exponent was -1.1608 to -0.9801, encompassing values close to -1 and supporting an approximately inverse dependence of reactivity on reactor period over the investigated interval. The coefficient a, however, had a wider 95% confidence interval of 225.31–505.50, reflecting both the small number of observations and its mathematical association with the extrapolated model response at $T = 1$ s, far outside the investigated period range.

Figure 4 illustrates the corresponding variation of reactivity with reactor period. Reactivity decreases nonlinearly with increasing reactor period, while the In-hour and PRCT curves remain closely spaced throughout the investigated interval. The fitted power-law model similarly follows the observed period–reactivity behaviour, demonstrating that the empirical relationship provides a compact representation of the reactor response within the analysed range.

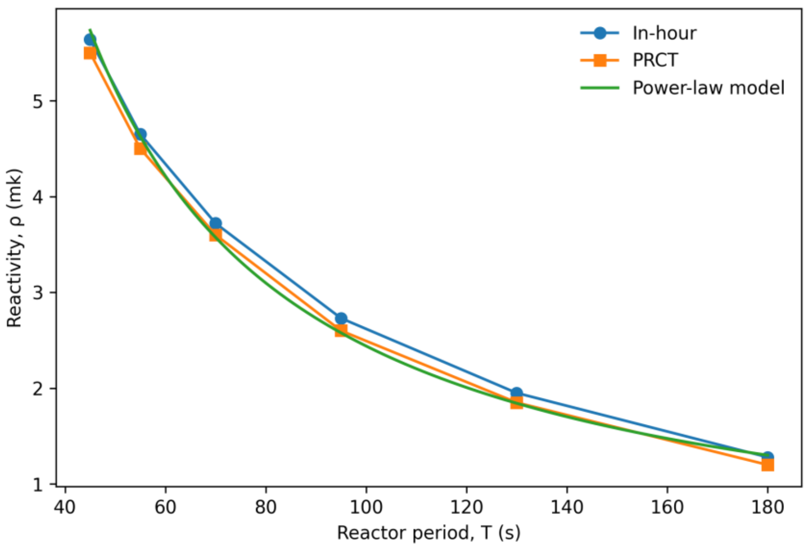

Figure 4. Reactor period–reactivity relationship for the LEU NIRR-1 showing In-hour and PRCT estimates and the fitted power-law model. The decrease in reactivity with increasing reactor period reflects the inverse dependence expected from point-reactor kinetics.

The exponent of approximately −1.07 has a physically meaningful interpretation because its negative sign describes the expected inverse relationship between reactor period and positive reactivity. In point-reactor kinetics, increasing positive reactivity increases the rate of neutron-population growth and consequently decreases the reactor period (Keepin, 1965; Lamarsh & Baratta, 2001). The observed power-law behaviour is therefore physically consistent with the established kinetics of a delayed-supercritical reactor.

Nevertheless, we did not regard the fitted exponent as a fundamental reactor-kinetics constant. The exact In-hour relationship is not a pure power law; rather, it consists of a prompt-neutron term together with contributions from delayed-neutron precursor groups having different decay constants (Keepin, 1965). The fitted relationship is therefore an empirical approximation to the In-hour behaviour over the restricted 45–180 s period range rather than an alternative theoretical formulation of reactor kinetics. This interpretation is particularly relevant to the LEU NIRR-1 because its neutronic characteristics have required renewed validation following conversion from HEU fuel. The IAEA conversion analyses demonstrated the need to reassess reactor-specific neutronic parameters following HEU-to-LEU conversion (IAEA, 2018), while Asuku et al. (2023) subsequently reported close correspondence between calculated and commissioning parameters for the as-built LEU NIRR-1. The present period–reactivity model complements these efforts by providing an empirical representation of period-dependent reactivity rather than a full-core neutronic model.

3.4 Log-Linear Behaviour and Predictive Validation

The power-law character of the reactor period–reactivity relationship was further examined through the logarithmic transformation of the model previously defined in Equation (7). The near-linear distribution of the transformed observations shown in Figure 5 supports the suitability of the power-law representation over the investigated reactor-period interval. The negative slope provides a quantitative expression of the inverse dependence of reactivity on reactor period and is consistent with the behaviour predicted by point-reactor kinetics (Keepin, 1965; Lamarsh & Baratta, 2001).

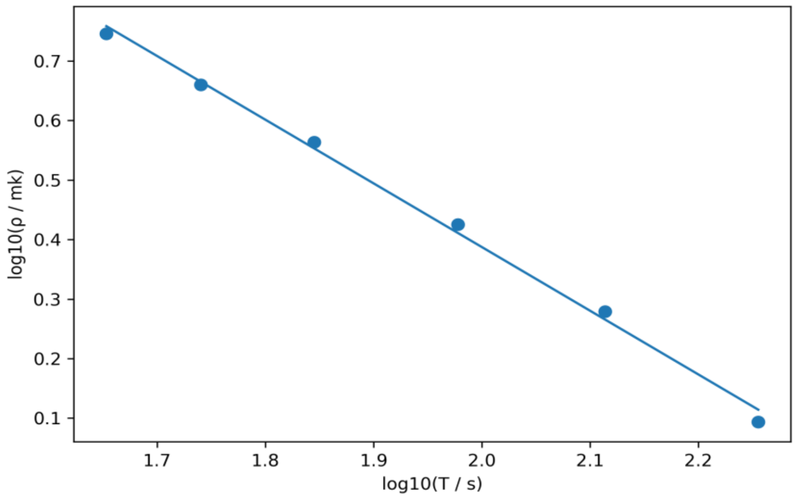

Figure 5. Log–log representation of the reactor period–reactivity relationship for the LEU NIRR-1. The approximately linear trend supports the power-law dependence of reactivity on reactor period, with the negative slope indicating decreasing reactivity as reactor period increases.

Model performance was further evaluated using both in-sample and out-of-sample error measures. Relative to the pair-mean reactivity, the fitted model produced an in-sample RMSE of 0.093 mk and MAE of 0.085 mk. Leave-one-out cross-validation (LOOCV) yielded an RMSE of 0.159 mk, MAE of 0.137 mk, and MAPE of 5.00%.

The increase in error from the in-sample RMSE of 0.093 mk to the cross-validated value of 0.159 mk is reasonable given that each LOOCV iteration uses only five observations for model fitting. More importantly, the cross-validated error remains relatively small compared with the magnitude of the measured reactivities and is comparable to the discrepancy observed between the In-hour and PRCT methods themselves. This suggests that the fitted relationship retains reasonable predictive stability when individual observations are omitted.

We suggest that the model should nevertheless be applied conservatively. Its validation is restricted to six observations spanning reactor periods of 45–180 s, and extrapolation beyond this interval is not presently supported by the available data. Consequently, the model is most appropriately regarded as an interpolation, consistency-checking, and reactor-physics analysis tool, rather than a replacement for the theoretically established In-hour equation, an approved facility PRCT, or reactor operating procedures (Keepin, 1965; IAEA, 2023).

3.5 Relationship to Previous NIRR-1 Reactor-Physics Studies

The present results form part of a continuing programme of reactor-specific characterization of NIRR-1. Jonah et al. (2007) developed and validated a detailed Monte Carlo model of the original HEU NIRR-1 core, providing an important computational foundation for subsequent reactor analyses. The later HEU-to-LEU conversion required renewed evaluation of criticality and other neutronic characteristics to demonstrate that the converted configuration satisfied operational and safety requirements (IAEA, 2018).

Following conversion, investigations have increasingly focused on validating the behaviour of the LEU core. Abubakar et al. (2022) analysed point-kinetics parameters, including effective delayed-neutron fraction, prompt-neutron lifetime, and mean neutron generation time, which are directly relevant to interpretation of period-based reactor kinetics. Ibrahim et al. (2022) reported close correspondence between experimental and analytical estimates of control-rod worth, while Asuku et al. (2023) demonstrated close agreement between calculated and commissioning neutronic parameters using an as-built computational representation of the LEU NIRR-1. These studies collectively reinforce the importance of reactor-specific cross-validation following changes in fuel enrichment and reactor configuration.

The present study extends this validation framework specifically to period-based reactivity conversion. Unlike earlier investigations primarily concerned with full-core neutronic modelling or control-rod-worth determination, the current analysis evaluates the internal consistency of the In-hour and PRCT methods when applied to an identical reactor-period dataset. Its additional contribution is the transparent reconstruction and predictive evaluation of a reactor-specific power-law relationship. Hence, it is important to emphasize again that this study should be viewed as a methodological cross-validation and reactor-kinetics modelling investigation, rather than as a new complete control-rod calibration experiment.

4. Conclusion

The In-hour equation and PRCT produced closely agreeing reactivity estimates for six LEU NIRR-1 reactor periods between 45 and 180 s. The In-hour values were systematically but only slightly higher than PRCT, with MAE = 0.120 mk, RMSE = 0.122 mk and mean absolute percentage deviation = 4.38%. Agreement regression yielded $R^2 = 0.99994$ and a slope close to unity, while exploratory Bland–Altman analysis quantified the positive bias at 0.120 mk. A reproducible reconstruction shows that the reported power law is obtained by fitting the arithmetic mean of the paired reactivity values: $\rho = 337.48T^{-1.070}$, commonly rounded to $\rho = 337.48T^{-1.07}$. The model gave $R^2 = 0.9963$ and LOOCV RMSE = 0.159 mk. These results support its use as a compact interpolation and cross-checking relation within the investigated 45–180 s range. The empirical model should not replace the In-hour equation, an approved PRCT or facility operating procedures. The strongest next step is independent validation with new low-power measurements, explicit uncertainty propagation and a larger number of rod positions.

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