Time Evolution of Cosmological Parameters in an FLRW Universe with an Empirical Dynamical Cosmological Term

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Abstract

The accelerated expansion of the Universe has motivated the study of dynamical cosmological models with a time-dependent cosmological term. In this work, we investigate the evolution of important cosmological parameters in a spatially flat Friedmann–Lemaître–Robertson–Walker (FLRW) Universe by introducing an empirical dynamical cosmological term expressed in terms of the Hubble parameter and its time derivative. Assuming a barotropic equation of state for the cosmic fluid, analytical expressions are obtained for the scale factor, Hubble parameter, deceleration parameter, energy density, gravitational constant, and cosmological term. Model parameters are constrained using the physical requirements of an expanding Universe, a decreasing Hubble parameter, and a transition from decelerated to accelerated expansion. The model predicts continuous expansion, a decreasing energy density, and a positive cosmological term that gradually decays with time. The gravitational constant also exhibits a time-dependent evolution, and its predicted present rate of change is consistent with previously reported observational limits. These results indicate that the proposed empirical dynamical cosmological term provides a physically viable framework for describing the evolution of the expanding Universe.

Keywords: FLRW cosmology, Cosmological parameters, Dynamical cosmological term, Time-varying gravitational constant, Cosmic acceleration, Deceleration parameter, Dark energy, Energy density.

1. Introduction

The study of the Universe and its evolution has remained one of the most important topics in modern cosmology. According to the standard cosmological model, the large-scale structure of the Universe can be described using the Friedmann–Lemaître–Robertson–Walker (FLRW) metric together with Einstein's field equations of General Relativity [1, 2]. These equations describe the dynamics of the Universe in terms of important cosmological quantities such as the scale factor, Hubble parameter, energy density, and cosmological constant.

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The cosmological constant, originally introduced by Einstein to obtain a static Universe [3], later gained renewed importance after observations of distant Type Ia supernovae revealed that the Universe is undergoing accelerated expansion [4, 5]. Subsequent observations of the cosmic microwave background radiation and large-scale structure further strengthened the evidence for a Universe dominated by dark energy [6–8]. The cosmological constant (Λ) is often considered the simplest explanation for dark energy; however, several theoretical issues, including the cosmological constant problem and coincidence problem, have motivated researchers to explore alternative possibilities [9, 10].

One important alternative is the idea of a time-dependent cosmological term or dynamical cosmological constant. Several authors have proposed models in which Λ varies with time or depends on cosmological quantities such as the Hubble parameter and scale factor [11–15]. Such models have been studied extensively to explain the late-time acceleration of the Universe and to provide better physical understanding of cosmic evolution. In many studies, a decaying cosmological term has been found to be compatible with observational data and capable of describing a transition from decelerated to accelerated expansion [16–18].

Another interesting possibility in cosmology is the consideration of a time-varying gravitational constant (G). The idea of variable G was first proposed by Dirac through his Large Number Hypothesis [19]. Since then, several studies have examined cosmological models involving variable G, as such models may help explain different stages of cosmic evolution [20–22]. Observational studies also provide constraints on the allowed variation of G, usually expressed through the quantity ($\dot{G}/G$) [23, 24]. Therefore, studying the simultaneous behaviour of Λ and G can provide useful insights into the dynamics of the Universe.

Several previous studies have shown that the Hubble parameter decreases with cosmic time, while the deceleration parameter changes from positive values in the early Universe to negative values at later times, indicating a transition from decelerated to accelerated expansion [25–27]. Similarly, models involving dynamical dark energy often predict a positive but gradually decreasing cosmological term [14, 16]. The decrease in energy density with cosmic expansion observed in many cosmological models is also consistent with the expected behaviour of an expanding Universe [2, 7].

In the present work, we investigate the time evolution of cosmological parameters by considering an empirical form of a dynamical cosmological term within the FLRW framework. A barotropic equation of state has been employed for the cosmic fluid. Analytical expressions for the scale factor, Hubble parameter, deceleration parameter, energy density, gravitational constant, and cosmological term have been obtained. Physical conditions such as positive expansion, decreasing Hubble parameter, and transition of the deceleration parameter from positive to negative values have been used to restrict the model parameters. The time evolution of these quantities has been analysed graphically, and the obtained results are compared with the findings of earlier studies wherever applicable.

2. Theory

2.1. Metric and Field Equations

The space-time geometry of the Universe in the FLRW framework is described by the following line element.

$$ds^2 = -c^2dt^2 + a(t)^2 \left[\frac{dr^2}{1 - Kr^2} + r^2(d\theta^2 + sin^2\theta \, d\phi^2)\right] \tag{1}$$

where $a(t)$ is the scale factor and K represents the spatial curvature.

Within the FLRW framework for flat space (i.e., $K = 0$), the dynamics of the Universe are governed by the following equations:

$$3H^2 = 8\pi G\rho + \Lambda \tag{2}$$

$$3(\dot{H} + H^2) = -4\pi G(\rho + 3p) + \Lambda \tag{3}$$

In equations (2) and (3), ρ and p represent the energy density and pressure of the cosmic fluid, respectively. The Hubble parameter $(\dot{a}/a)$ is denoted here by the symbol H. Λ is known as the cosmological constant, and G denotes the gravitational constant. A time-dependent Λ is often referred to as *cosmological term*, instead of *cosmological constant*.

To obtain these equations, the FLRW metric (eqn. 1) has been combined with Einstein's field equations of General Relativity (eqn. 4), and we have taken $c = 1$ for these calculations.

$$R_{ij} - \frac{1}{2}Rg_{ij} = -\frac{8\pi GT_{ij}}{c^4} + \Lambda g_{ij} \tag{4}$$

For the present study, we have used the barotropic equation of state which is given by,

$$p = \omega\rho \tag{5}$$

Here ω is known as the equation-of-state (EoS) parameter. Substituting equation (5) into equation (3) and then combining it with equation (2), we get the following equation.

$$2\dot{H} = (1 + \omega)(\Lambda - 3H^2) \tag{6}$$

Equation (6) is a differential equation which can be solved to find the expression for the scale factor (a), by using an empirical expression for Λ in terms of ω and H.

Using equations (2), (3) and (5), one obtains the following expression for the gravitational constant (G).

$$G = \frac{-\dot{H}}{4\pi\rho(1 + \omega)} \tag{7}$$

The equation of conservation of the cosmic fluid, under the barotropic equation of state ($p = \omega\rho$), is given by,

$$\frac{d\rho}{dt} + 3H\rho(1 + \omega) = 0 \tag{8}$$

Solving equation (8), one obtains the following expression for ρ.

$$\rho = \rho_0 \left(\frac{a}{a_0}\right)^{-3(\omega+1)} \tag{9}$$

where a_0 and ρ_0 are the values of the scale factor and energy density at present time (t_0). Here t_0 denotes the age of the universe, which is approximately 13.8 billion years or 4.35×10^{17} seconds.

2.2. An Empirical Expression for a Dynamical Cosmological Term (Λ)

For the present study, we have used the following empirical expression for the cosmological term (Λ).

$$\Lambda = 3H^2 + AH^k t^l \tag{10}$$

where k and l are constant quantities.

Substituting equation (10) into equation (6), one gets,

$$2\dot{H} = (1+\omega)AH^k t^l \tag{11}$$

Solving equation (11), for $k \neq 1, l \neq -1$, one gets,

$$H = \left[\frac{A(1+\omega)(1-k)}{2(l+1)} t^{l+1} + C_1\right]^{\frac{1}{1-k}} \tag{12}$$

Here, C_1 is a constant of integration. Taking $C_1 = 0$ for analytical convenience, one gets the following expression for the scale factor (a) from equation (12).

$$a(t) = \exp\left[\left\{\frac{A(1+\omega)}{2(1+l)}\right\}^{\frac{1}{1-k}} \frac{(1-k)^{\frac{2-k}{1-k}}}{2+l-k} t^{\frac{l-k+2}{1-k}} + C_2\right] \tag{13}$$

Here, C_2 is a constant of integration.

The choice $C_1 = 0$ is adopted to obtain an analytically tractable solution; more general solutions with $C_1 \neq 0$ may be investigated separately.

Using equation (13), the deceleration parameter (q) is obtained as,

$$q(t) = -1 - \frac{A(1+\omega)}{2}\left[\frac{A(1+\omega)(1-k)}{2(l+1)}\right]^{\frac{k-2}{1-k}} t^{-\left(\frac{l-k+2}{1-k}\right)} \tag{14}$$

Based on the cosmological requirements, such as: 1) $H > 0$, 2) $dH/dt < 0$, 3) q decreases from positive to negative with time (indicating transition from deceleration to acceleration), along with the incorporation of facts that, $a = a_0$, $H = H_0$, $q = q_0$, at $t = t_0$, one gets the following three equations.

$$a(t) = a_0\, Exp\left[\frac{H_0 t_0}{n+1}\left\{\left(\frac{t}{t_0}\right)^{n+1} - 1\right\}\right] \tag{15}$$

$$H(t) = H_0 \left(\frac{t}{t_0}\right)^n \tag{16}$$

$$q(t) = -1 - \frac{n}{H_0 t_0 \left(\frac{t}{t_0}\right)^{n+1}} \tag{17}$$

where, $n = \frac{l+1}{1-k}$, with $-1 < n < 0$ for satisfying the above-mentioned requirements.

Based on equation (13), one obtains, $C_2 = \ln a_0 - \frac{H_0 t_0}{n+1}$, which has been used to obtain equation (15) from equation (13).

Substituting equation (15) into equation (9), we get,

$$\rho(t) = \rho_0 \, Exp\left[-3(\omega+1)\frac{H_0 t_0}{n+1}\left\{\left(\frac{t}{t_0}\right)^{n+1} - 1\right\}\right] \tag{18}$$

Substituting equations (16) and (18) into equation (7), one gets,

$$G(t) = G_0 \left(\frac{t}{t_0}\right)^{n-1} Exp\left[3(\omega+1)\frac{H_0 t_0}{n+1}\left\{\left(\frac{t}{t_0}\right)^{n+1} - 1\right\}\right] \tag{19}$$

Using the fact that $q = q_0$ at $t = t_0$ in equation (17), one obtains the following expression for n.

$$n = -(q_0 + 1) H_0 t_0 \tag{20}$$

Using equation (20), one gets, $n = -0.4581$, based on the values of the parameters q_0, H_0, t_0 in SI units, with q_0 being dimensionless. In this article, we have used the following values of cosmological parameters at the present time (i.e., at $t = t_0 = 4.35 \times 10^{17} sec$): $H_0 = 72.20\ km\ s^{-1} Mpc^{-1} = 2.34 \times 10^{-18} sec^{-1}$, $q_0 = -0.55$, $\rho_0 = 9.90 \times 10^{-27}\ kg\ m^{-3}$, $G_0 = 6.67 \times 10^{-11}\ m^3\ kg^{-1}\ s^{-2}$.

Substituting equation (20) into equation (17), we obtain,

$$q(t) = -1 + (q_0 + 1)\left(\frac{t}{t_0}\right)^{-(n+1)} \tag{21}$$

Since the present model requires an expanding universe ($H > 0$), a decreasing Hubble parameter ($dH/dt < 0$), and a transition of the deceleration parameter (q) from positive to negative values with time, one obtains the following condition for n: $-1 < n < 0$. Substituting equations (16), (18), (19) into equation (7), one gets, $n = -4\pi(1+\omega)\frac{G_0 \rho_0 t_0}{H_0} = -1.58\ (1 + \omega)$, using the parameter values in SI units. Now, using the requirement for n, i.e., $-1 < n < 0$, one obtains, $-1 < \omega < -0.367$.

As per equation (3), we have, $\ddot{a} = -\frac{4\pi G \rho a}{3}(1 + 3\omega) + \frac{\Lambda a}{3}$. To obtain an approximate fluid-dominated acceleration criterion (i.e., $\ddot{a} > 0$), we consider the limiting case of negligible Λ, which yields $\rho + 3\omega\rho < 0$, giving $\omega < -1/3$, which is another upper limit for ω which is sufficiently close to the previously obtained one (which is -0.367). Using the barotropic relation $p = \omega\rho$, the inequality representing acceleration becomes $\rho + 3p < 0$.

Substituting equation (16) into equation (10), one obtains the following expression for the cosmological term $\Lambda(t)$, where, $n = \frac{l+1}{1-k}$.

$$\Lambda(t) = 3kH_0{}^2\left(\frac{t}{t_0}\right)^{2n} + \frac{nH_0\, l}{t_0}\left(\frac{t}{t_0}\right)^{n-1} \tag{22}$$

Based on equation (16), equation (22) can be expressed as,

$$\Lambda = 3kH^2 + l\,\dot{H} \tag{23}$$

Dependence of Λ on the Hubble parameter and its derivatives has been widely explored in phenomenological cosmology because such quantities naturally encode the expansion history of the Universe.

The present accelerated expansion of the Universe requires a positive cosmological term ($\Lambda > 0$). Since $H^2 > 0$, one must choose $k > 0$ to keep the first term of equation (23) positive. Further, under the condition of $-1 < n < 0$, where $n = \frac{l+1}{1-k}$, choosing $k < 1$ implies $k - 2 < l < -1$. Since $\dot{H} < 0$, the term $l\,\dot{H}$ thus remains positive. Thus we restrict the parameter k to the interval $0 < k < 1$. Hence the admissible interval obtained from the positivity of $\Lambda(t)$ is consistent with the previously obtained physical restriction $-1 < n < 0$. Within this range, the cosmological term is expected to remain physically viable while exhibiting a decaying behaviour with cosmic time.

Using the relation $n = \frac{l+1}{1-k}$, one obtains, $l = n(1-k) - 1 = -0.458(1-k) - 1$. One may vary k in the interval $0 < k < 1$, and find the corresponding l using this relation. Based on this relation, equation (23) takes the following form.

$$\Lambda = 3kH^2 + \{-0.458(1-k) - 1\}\,\dot{H} \tag{24}$$

Using equations (16) and (19), one obtains,

$$\frac{\dot{G}}{G} = \frac{n-1}{t} + 3(1+\omega)H(t) \tag{25}$$

The value of $\left(\frac{\dot{G}}{G}\right)_{t=t_0}$, as obtained from equations (20) and (25) is given below.

$$\left(\frac{\dot{G}}{G}\right)_{t=t_0} = -\frac{1}{t_0} - H_0(1+q_0) + 3(1+\omega)H_0 \tag{26}$$

The allowed parameter ranges used in all numerical plots are: $-1 < n < 0$, $-1 < \omega < -0.367$, $0 < k < 1$.

3. Results and Discussion

Figure 1 shows the time-variation of the scale factor (a) using equation (15). The curve shows that it increases with time with a gradually increasing slope.

Figure 2 shows the time dependence of the Hubble parameter (H) based on equation (16). It is positive and decreases with time.

Figure 3 shows the time evolution of the deceleration parameter (q), based on equation (21). It shows a transition from a phase of deceleration ($q > 0$) to a phase of acceleration ($q < 0$) of cosmic expansion.

For these plots, the value of n is obtained from equation (20), and this value is -0.458.

Figure 4 shows the variation of the energy density (ρ) as a function of time, based on equation (18), for three values of ω. It decreases with time at a gradually decreasing rate. We see here a faster decrease in energy density for less negative values of ω.

Figures 5 and 6 show the time-variation of the gravitational constant (G) in normal scale and log scale respectively, for three values of ω, based on equation (19). The log-scale plots show the behaviour of $G(t)$ more distinctly as t/t_0 increases, in comparison to the plots with normal scale. These plots show that G initially decreases and then increases with time, with faster change for less negative values of ω. The late-time increase is most clearly visible in the logarithmic representation (Figure 6). The late-time increase in $G(t)$ arises from the competition between the power-law decay term and the exponential factor in equation (19). The phase of increasing G begins later for more negative values of ω. The late-time increase of G is a model prediction arising from the adopted empirical form of the cosmological term and may be tested against future observational constraints.

Figure 7 shows the variation of the cosmological term (Λ) as a function of time for three values of the parameter k. It is positive and decreases with time with a gradually decreasing slope. A faster rate of change is observed for smaller values of k. The parameters, n, k and l, are connected by the relation $n = \frac{l+1}{1-k}$, where the value of n is given by equation (20). The unit of Λ, shown as s^{-2} in Figure 7, is consistent with equation (23).

Figure 8 shows the time evolution of $\dot{G}/G$ for three values of ω. It changes faster for smaller negative values of ω. It rises from a high negative value toward zero, with a gradually decreasing rate of change. The inset shows the behaviour over a short span close to the present time (i.e., $t = t_0$). The quantity $\dot{G}/G$ increases with time. For $\omega = -0.65$ and $\omega = -0.90$, it rises from large negative values toward zero, whereas for $\omega = -0.40$ it eventually becomes positive near the present epoch (i.e., $t = t_0$).

It is important to calculate $\left(\frac{\dot{G}}{G}\right)_{t=t_0}$ which is the relative rate of change of G at the present time ($t = t_0$). Based on our model, we have determined the values of $\left(\frac{\dot{G}}{G}\right)_{t=t_0}$ for the three values of ω shown in Figure 8 (in the interval of $-1 < \omega < -0.367$). These values are:

1) for $\omega = -0.40$, $\left(\frac{\dot{G}}{G}\right)_{t=t_0} = +8.60 \times 10^{-19}\ s^{-1} = +2.71 \times 10^{-11}\ year^{-1}$, 2) for $\omega = -0.65$, $\left(\frac{\dot{G}}{G}\right)_{t=t_0} = -8.95 \times 10^{-19}\ s^{-1} = -2.82 \times 10^{-11}\ year^{-1}$, 3) for $\omega = -0.90$, $\left(\frac{\dot{G}}{G}\right)_{t=t_0} = -2.65 \times 10^{-18}\ s^{-1} = -8.36 \times 10^{-11}\ year^{-1}$. These values lie within the order of magnitude of the observational limits reported in Refs. [23, 24].

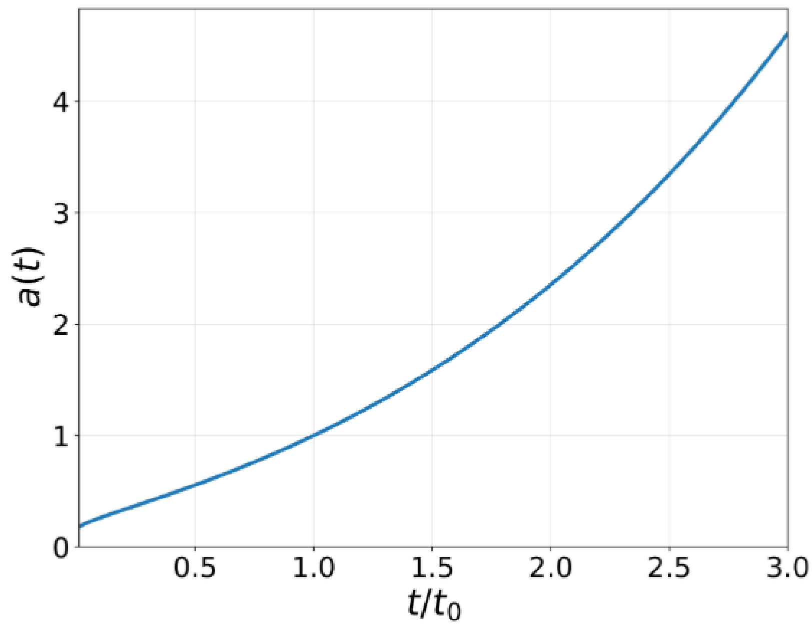

Figure 1: Plot of scale factor (a) as a function of time.

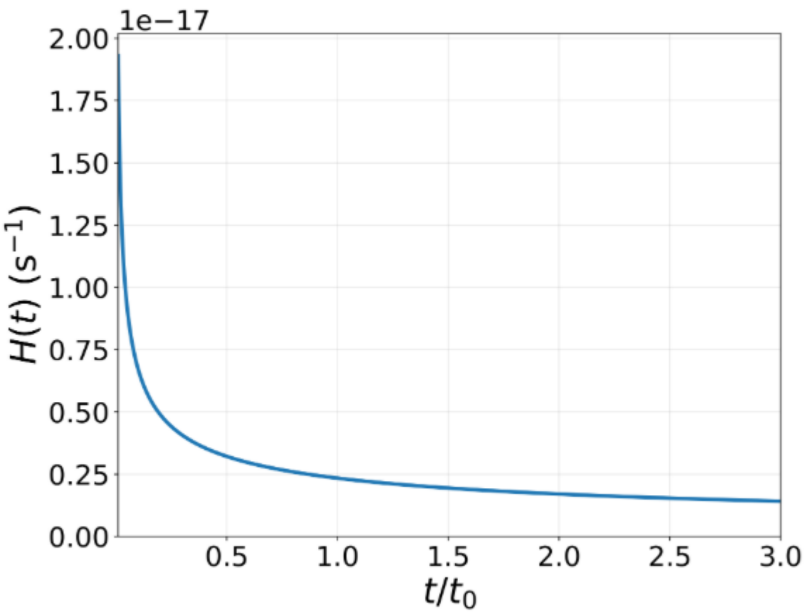

Figure 2: Plot of Hubble parameter (H) as a function of time.

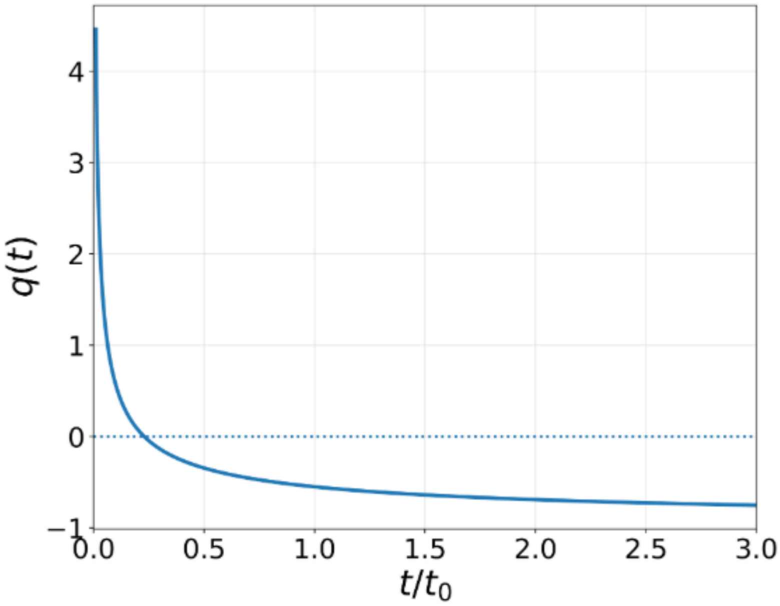

Figure 3: Plot of deceleration parameter (q) as a function of time.

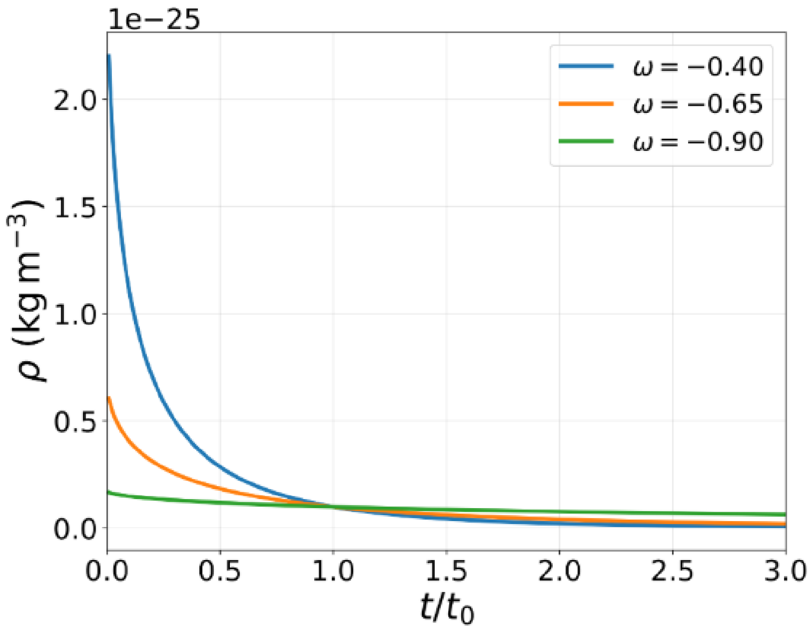

Figure 4: Plot of energy density (ρ) as a function of time for three values of the EoS parameter (ω).

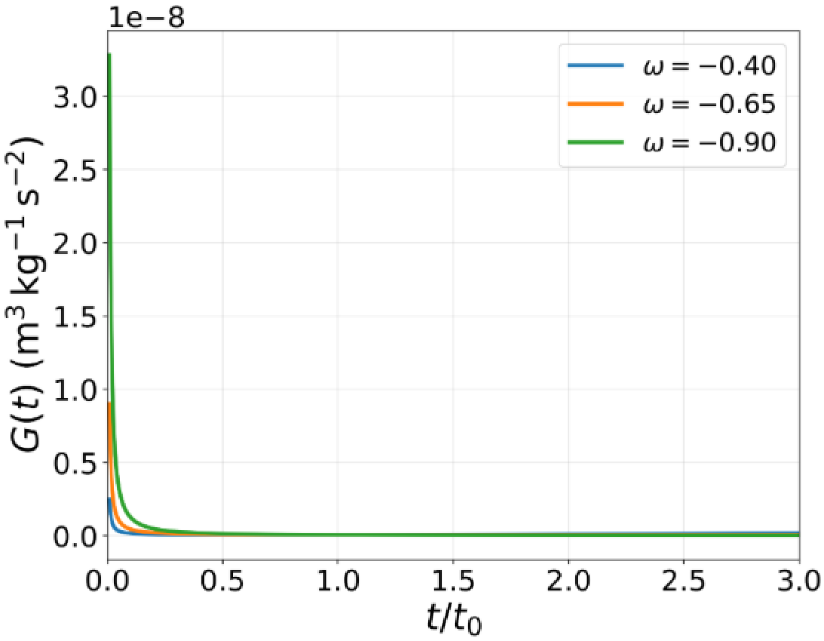

Figure 5: Plot of gravitational constant (G) as a function of time for three values of the EoS parameter (ω).

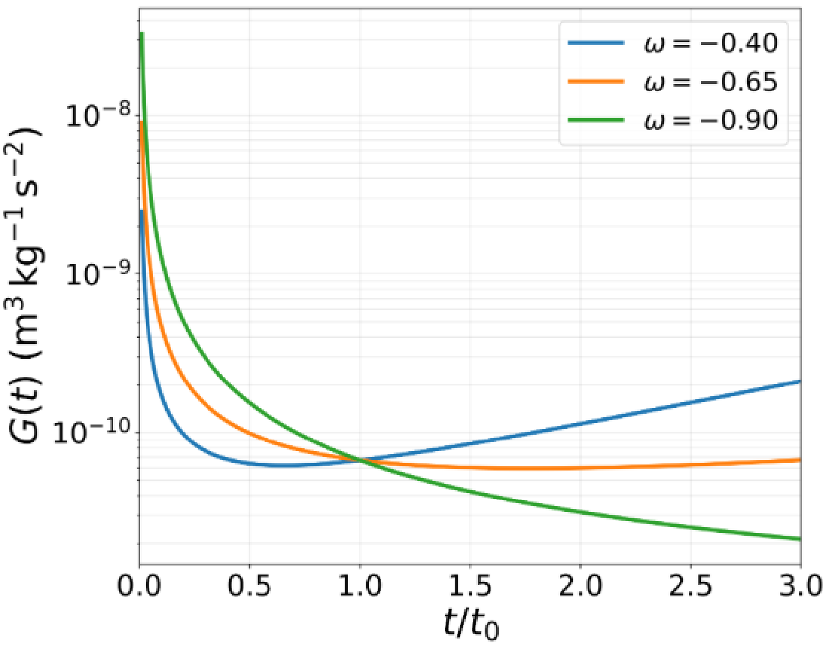

Figure 6: Plot of gravitational constant (G) (in log scale) as a function of time for three values of the EoS parameter (ω).

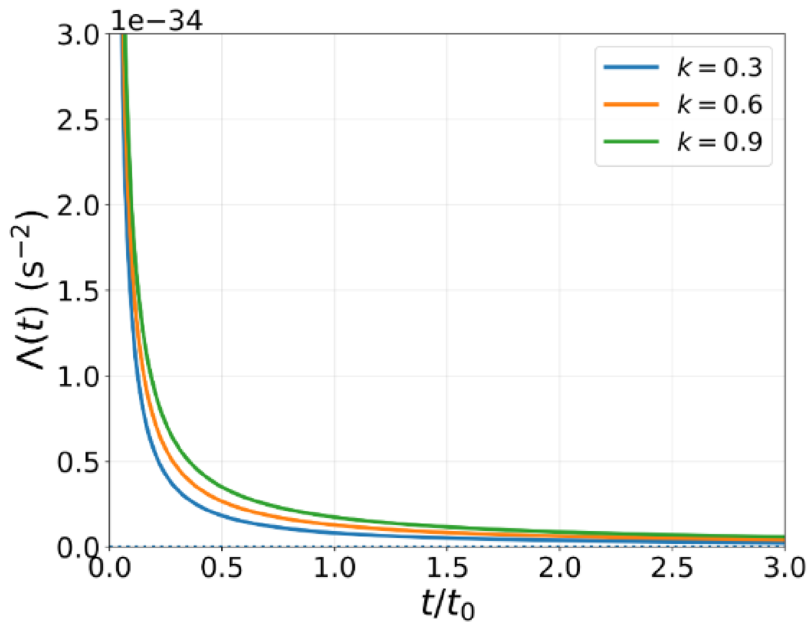

Figure 7: Plot of the cosmological term $\Lambda(\mathrm{t})$ as a function of time for three values of the parameter k. The unit corresponds to equation (23).

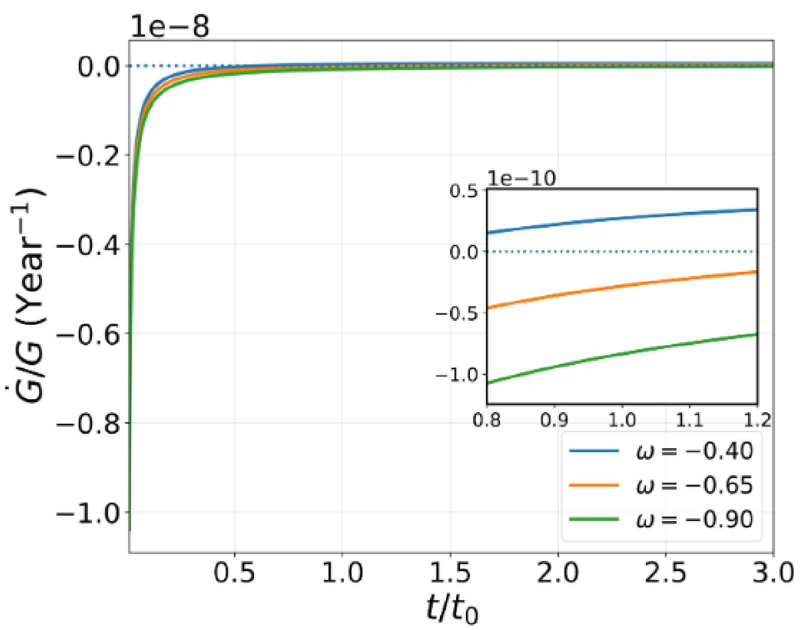

Figure 8: Plot of $\dot{G}/G$ as a function of time for three values of ω. The inset shows the behaviour over a short span close to the present time ($t = t_0$).

4. Conclusion

In the present work, we have studied the time evolution of important cosmological parameters by considering an empirical form of a dynamical cosmological term within the framework of the FLRW Universe model. A barotropic equation of state has been used to describe the cosmic fluid, and analytical expressions for the scale factor, Hubble parameter, deceleration parameter, energy density, gravitational constant, and cosmological term have been obtained.

The obtained results show that the scale factor increases continuously with time, indicating an expanding Universe. The Hubble parameter remains positive and decreases with time, which agrees with the expected behaviour of an expanding cosmological model and is consistent with several observational and theoretical studies [4–8, 25]. The deceleration parameter exhibits a transition from positive to negative values, indicating a change from an early decelerating phase to the present accelerating phase of expansion. Such a transition has also been reported in different cosmological investigations and observational studies related to dark energy and late-time acceleration [4–8, 16–18].

The energy density of the cosmic fluid decreases with time, which is physically expected in an expanding Universe. The cosmological term remains positive and gradually decreases with cosmic time, supporting the possibility of a decaying dark energy component, which has been discussed in several dynamical cosmological constant models [11–18]. The behaviour of the cosmological term obtained in the present study is therefore found to be qualitatively similar to earlier theoretical investigations.

The gravitational constant exhibits a time-dependent evolution in the present model. The obtained values of $\dot{G}/G$ are comparable to observational limits reported in earlier studies of variable-G cosmologies [23, 24]. This suggests that the proposed model remains physically viable from an observational point of view.

Overall, the present empirical form of the dynamical cosmological term provides a physically acceptable description of cosmic evolution and satisfies important cosmological requirements, such as positive expansion, decreasing Hubble parameter, and transition to accelerated expansion. Since the obtained results show qualitative agreement with several earlier observational and theoretical studies, the present model may serve as a useful framework for studying the evolution of cosmological parameters in an expanding Universe.

The present analysis is restricted to a spatially flat FLRW Universe and a specific empirical form of the cosmological term; more general forms may be investigated in future work. Future studies may include observational fitting and comparison with cosmological datasets.

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