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A Study of Time Dependence of Cosmological Parameters for a Dynamical Cosmological Constant Using a Hybrid Scale Factor

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Abstract

In the present study we have determined the time dependence of some cosmological parameters of an isotropic and homogeneous universe, based on an empirical scale factor. The scale factor has been chosen in a manner such that the deceleration parameter changes sign from positive to negative as time goes on. This behaviour of the scale factor is consistent with the transition of the state of the universe from a decelerated expansion to its present state of accelerated expansion. This empirical scale factor is a product of exponential and power-law functions of time, which is often referred to as the *hybrid scale factor*. First part of the study is based on a constant *equation of state* (EoS) parameter and its second part is based on a time-varying EoS parameter. The gravitational constant is shown by this model to be decreasing with time. The cosmological constant (denoted by lambda), is found in the present study to be increasing with time, with a gradually decreasing rate of change. In the early universe it rises very steeply from a negative value to a less negative value at the present time, and then, as per predictions by this model, it is likely to increase very slowly to reach a positive value in future. All our theoretical findings have been depicted here graphically.

Keywords: Hybrid Scale Factor, FLRW Universe, Dynamical Cosmological Term (Λ), Dynamical EoS Parameter (ω), Dynamical Gravitational Term (G).

1. Introduction

The nature of time-variation of cosmological quantities has always been a central focus in the field of cosmology. However, in the past few years, this topic has gained much interest, causing it to attract a huge lot of attention from the entire community of researchers. As widely accepted, a cosmological model is a mathematical explanation of the entire cosmos [1]. The study of various categories of cosmological models tries to explain the reasons for their current aspects along with the nature of evolution over time [1].

Homogeneity and isotropy are the structural consequences of the cosmological principle that may be evaluated the most [2]. Both homogeneity and isotropy imply that the cosmos appears the same at every point in space and that the universe appears the same in all directions,

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respectively [2]. One of the most recent and notable advancement in contemporary cosmology is the finding of the cosmic acceleration [3, 4]. Understanding the physical cause of cosmic acceleration is still a challenging endeavour [5]. Based on cosmological observations throughout the world, it has been convincingly established that the universe undergoes a process of expansion with acceleration [6, 7]. Research is going on extensively to understand the nature of the agent causing an accelerated expansion [5].

If gravitation had been the only interaction governing the motions of celestial bodies, the expansion of the universe would have continued with deceleration [6]. On the basis of the observational findings from supernova 1a, it was concluded that there is a negative pressure generated by an exotic form of energy, referred to as dark energy (DE), which is considered to be responsible for the present phase of accelerated expansion of the universe [3, 4]. The functioning of this mysterious DE can only be determined by extensive investigations [5]. A thorough analysis of supernova data has led to an inference that the universe has changed its phase from decelerated expansion to accelerated expansion, resulting in the change of sign of the deceleration parameter from positive to negative [3, 4].

In the vast scientific literature regarding investigations to find the nature of cosmic acceleration, one generally finds approaches through mainly two ways [5]. One of these ways is to construct mathematical models using modified theories of gravity (which are based on modifications of Einstein's theory of general relativity) and explore their characteristics [7, 8]. The other way is to investigate the cosmological observations by formulating dark energy models [5]. A parameter, named cosmological constant (denoted by Λ), has been said to be representing DE in lots of models on cosmology [9]. There are various dark energy models in scientific literature, namely quintessence, phantom, k-essence and quintom [5, 10-12]. The parameter Λ was initially used in Einstein's theory as a time-independent parameter [6], but it is presently looked upon as a time-varying quantity due to certain shortcomings corresponding to Coincidence Problem and Cosmological Constant Problem [9, 13].

In the present investigation, we have examined the evolution (with cosmic time) of several cosmological quantities, namely the scale factor (a), Hubble parameter (H), deceleration parameter (q), and the energy density of the universe (ρ), together with the temporal behavior of the gravitational constant (G), the cosmological constant (Λ), and the fractional rate of change of the gravitational constant ($\dot{G}/G$). This analysis has been carried out with the help of an empirical form of the scale factor given by $a = Ae^{\alpha t}t^{\beta}$, where α and β are constants. Explicit time-dependent expressions for all the above-mentioned parameters have been derived, and their time-evolution has been graphically depicted for i) a constant equation-of-state parameter (ω), and ii) a time-varying equation-of-state parameter [$\omega(t)$].

2. Equations Governing the Cosmological Model

Current research indicates that the universe is in a paramount era of *dark energy*, where this enigmatic component dominates and drives the observed acceleration [9, 14]. Initially, Einstein introduced the cosmological constant (Λ) in 1917 to achieve a static universe model. However, Edwin Hubble's 1929 discovery of galactic red-shift provided evidence for an expanding universe, prompting a revision of this static view [15]. Furthermore, observations suggest that the cosmological constant (Λ), and potentially the gravitational constant (G), may not be constant over time [16-18]. The accelerated expansion of the universe lends credence to a positive and potentially time-varying Λ [19, 20]. In response to these findings, theories like Berman's, based on conservation laws, attempt to incorporate the variability of G and Λ into the field equations [21, 22].

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = -8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu} \tag{01}$$

where G and Λ varies with time.

It is known that, the FLRW (Friedmann-Lemaitre-Robertson-Walker) metric mathematically represents the characteristics of the space-time geometry based on the cosmological principle. According to the cosmological principle, at very large cosmic distances, our Universe is found to have both homogeneity and isotropy. The expression for the FLRW metric is given by,

$$ds^2 = c^2 dt^2 - a^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + sin^2\theta\, d\phi^2)\right] \tag{02}$$

where K denotes the spatial curvature. For various values of $K = -1,0,1$ we can define an open, flat, and closed universe, respectively. We have used $K = 0$ in this model.

This can lead to the equations,

$$-\frac{2\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} - \frac{K}{a^2} = 8\pi G p - \Lambda \tag{03}$$

and,

$$3\frac{\dot{a}^2}{a^2} + \frac{3K}{a^2} = 8\pi G \rho + \Lambda \tag{04}$$

From equation (3) and (4) we can find the corresponding equations of G and Λ for different scale factors. An over-dot represents the derivative with respect to cosmic time t.

On addition of equation (3) and (4), we get,

$$-\frac{2\ddot{a}}{a} + 2\frac{\dot{a}^2}{a^2} + \frac{2K}{a^2} = 8\pi G(p + \rho) \tag{05}$$

Differentiating equation (4) with respect to time and by adding this to equation (5), we have

$$3\frac{\dot{a}}{a} \times 8\pi G(\rho + p) + 8\pi G\dot{\rho} + 8\pi \dot{G}\rho + \dot{\Lambda} = 0 \tag{06}$$

Based on equation (6) one may write the following two equations.

$$8\pi \dot{G}\rho + \dot{\Lambda} = 0 \tag{07}$$

$$\frac{3\dot{a}}{a} 8\pi G(p + \rho) + 8\pi G\dot{\rho} = 0 \tag{08}$$

In the analysis of physically relevant cosmological models, two key observational quantities are the Hubble parameter, $H(t)$, and the deceleration parameter, $q(t)$. The Hubble parameter, defined as $\mathrm{H(t)} = \frac{\dot{a}}{a}$ where $a(t)$ is the scale factor and $\dot{a}$ its time derivative, quantifies the instantaneous rate of expansion of the universe. The deceleration parameter, defined as $q(t) = -\frac{\ddot{a}a}{\dot{a}^2}$ where, $\ddot{a}$ is the second order time derivative of the scale factor, characterizes the temporal evolution of the expansion rate, specifically indicating whether the expansion is accelerated ($q < 0$) or decelerated ($q > 0$). These parameters are essential for constraining cosmological models and interpreting observational data [23-25].

Considering $\frac{\dot{a}}{a}$ as H we can rewrite the equation (8) as,

$$\dot{\rho} + 3H(p + \rho) = 0 \tag{09}$$

Based on equation (9) one gets,

$$\frac{d}{dt}(\rho a^3) = -3pa^2\dot{a} \tag{10}$$

For the formulation of the isotropic and homogeneous model, we have used the following relation between ρ and p.

$$p = \omega\rho \tag{11}$$

Here, the symbol ω denotes the *equation of state* (EoS) parameter, which is not necessarily a constant. When ω becomes zero, one gets the pressure-less model ($p = 0$) and if ω acquires the non-negative value $\omega = 1/3$, then the model represents the radiation dominated era, as suggested by scientific literatures [26-28]. As per SN Ia data we have $-1.67 < \omega < -0.6237$ and the range obtained by a combination of galaxy clustering statistics and SNIa data (with CMB anisotropy) is $-1.33 < \omega < 0.79$ [29-30].

Integrating equation (9) and using equation (11) we get,

$$\rho = ca^{-3(\omega+1)} \tag{12}$$

For $a = a_0$ we have $\rho = \rho_0$ and therefore,

$$\rho = \rho_0 \left(\frac{a_0}{a}\right)^{3(\omega+1)} \tag{13}$$

Using equation (11), equation (5) can be rewritten as,

$$-\frac{2\ddot{a}}{a} + \frac{2\dot{a}^2}{a^2} + \frac{2K}{a^2} = 8\pi G\rho(\omega + 1) \tag{14}$$

Solving equation (14), and using equation (13) we get,

$$G = \frac{-\frac{2\ddot{a}}{a} + \frac{2\dot{a}^2}{a^2} + \frac{2K}{a^2}}{8\pi(\omega+1)\rho_0\left(\frac{a_0}{a}\right)^{3(\omega+1)}} \tag{15}$$

Putting $\frac{\ddot{a}}{a} = -\frac{\ddot{a}a}{\dot{a}^2}\left(-\frac{\dot{a}^2}{a^2}\right) = -qH^2$ in equation (15) we get,

$$G = \frac{qH^2 + H^2 + \frac{K}{a^2}}{4\pi(\omega+1)\rho_0\left(\frac{a_0}{a}\right)^{3(\omega+1)}} \tag{16}$$

From equation (4) we get,

$$\Lambda = 3H^2 + \frac{3K}{a^2} - 8\pi G\rho \tag{17}$$

simplifying equation (17) by putting the expressions of ρ and G from equation (13) and (16), we get,

$$\Lambda = 3H^2 + \frac{3K}{a^2} - \frac{2\left(qH^2 + H^2 + \frac{K}{a^2}\right)}{\omega+1} \tag{18}$$

3. Time Dependence of H, q, ρ, G, Λ based on an Empirical Scale Factor

The scale factor used for the present study is,

$$a = Ae^{\alpha t}t^{\beta} \tag{19}$$

The reason for choosing this scale factor is that, it generates a deceleration parameter which changes sign from positive to negative as a function of time, indicating the transition of the universe from a phase of decelerated expansion to its present phase of accelerated expansion, as has been obtained from recent astrophysical studies.

Considering Equation of state parameter ω as constant, we can get,

Hubble parameter based on the scale factor of equation (19) is given by,

$$H = \frac{\dot{a}}{a} = \alpha + \frac{\beta}{t} \tag{20}$$

Where α , β are arbitrary constants.

and, the deceleration parameter is given by,

$$q = \frac{\frac{\beta}{t^2}}{\left(\alpha + \frac{\beta}{t}\right)^2} - 1 \tag{21}$$

Solving equations (20) & (21) we get the values for the constants, $\alpha = H_0 - q_0 t_0 H_0^2 - t_0 H_0^2$ and $\beta = H_0^2 t_0^2 (1 + q_0)$, using the fact that $H = H_0$ and $q = q_0$ at $t = t_0$.

Therefore, equation (20) and (21) can be rewritten as,

$$H = H_0 - q_0 t_0 H_0^2 - t_0 H_0^2 + \frac{H_0^2 t_0^2 (1+q_0)}{t} \tag{22}$$

and,

$$q = \frac{H_0^2 t_0^2 (1+q_0)}{\left[\left(H_0 - q_0 t_0 H_0^2 - t_0 H_0^2\right)t + H_0^2 t_0^2 (1+q_0)\right]^2} - 1 \tag{23}$$

where H_0, q_0 are the present values of the Hubble parameter and the deceleration parameter respectively, and t_0 is the age of the universe. $t_0 = 4.32 \times 10^{17}\ sec$.

Based on recent scientific literature [31-37], we have obtained the following values of Hubble parameter, deceleration parameter and energy density: $H_0 = 72.20\ km\ s^{-1}Mpc^{-1} = 2.27 \times 10^{-18} sec^{-1}, q_0 = -0.55, \rho_0 = 9.83 \times 10^{27}\ kg\ m^{-3}$ where ρ_0 is the energy density of the universe at present time. Using the values of H_0, q_0 and t_0, one gets $\alpha = 1.27 \times 10^{-18}$ and $\beta = 4.33 \times 10^{-1}$.

Recalling equation (13) and putting the expressions for a_0, a, α, β we get the time dependence of ρ as follows.

$$\rho = \rho_0 \left[e^{(H_0 - q_0 t_0 H_0^2 - t_0 H_0^2)(t_0 - t)} \left(\frac{t_0}{t}\right)^{q_0 t_0^2 H_0^2 + t_0^2 H_0^2} \right]^{3(\omega+1)} \tag{24}$$

From the equation (16) we get,

$$G = \frac{qH^2 + H^2 + \frac{k}{a^2}}{4\pi(\omega+1)\rho} \tag{25}$$

Substituting for the expressions of all parameters present in equation (25), we get,

$$G = \frac{\frac{q_0 t_0^2 H_0^2 + t_0^2 H_0^2}{t^2} + \frac{k}{A^2 e^{2(H_0 - q_0 t_0 H_0^2 - t_0 H_0^2)t}\, t^{2(q_0 t_0^2 H_0^2 + t_0^2 H_0^2)}}}{4\pi(\omega+1)\rho_0 \left[e^{3(H_0 - q_0 t_0 H_0^2 - t_0 H_0^2)(t_0 - t)(\omega+1)} \left(\frac{t_0}{t}\right)^{3(q_0 t_0^2 H_0^2 + t_0^2 H_0^2)(\omega+1)} \right]} \tag{26}$$

Now, applying the boundary condition that $a = a_0$ at $t = t_0$, in the expression for the scale factor, one gets the following expression for the constant A.

$$A = \frac{a_0^2}{e^{2\alpha_1 t_0}\, t_0^{2\beta_1}} \tag{27}$$

Rewriting the time dependent expression of G by putting the expression for A, one gets,

$$G = \frac{\frac{q_0 t_0^2 H_0^2 + t_0^2 H_0^2}{t^2} + \frac{k}{a_0^2 e^{2(H_0 - q_0 t_0 H_0^2 - t_0 H_0^2)(t - t_0)} \left(\frac{t}{t_0}\right)^{2(q_0 t_0^2 H_0^2 + t_0^2 H_0^2)}}}{4\pi(\omega+1)\rho_0 \left[e^{3(H_0 - q_0 t_0 H_0^2 - t_0 H_0^2)(t_0 - t)(\omega+1)} \left(\frac{t_0}{t}\right)^{3(q_0 t_0^2 H_0^2 + t_0^2 H_0^2)(\omega+1)} \right]} \tag{28}$$

We have found that, to get $G = G_0$ correctly at $t = t_0$, from equation (28), one needs to use $\omega = -0.722$. The present value of the gravitational constant, $G_0 = 6.67 \times 10^{-11} Nm^2 Kg^{-2}$.

Using equation (17), the time dependent expression for Λ becomes,

$$\Lambda = 3\left[(H_0 - q_0 t_0 H_0^2 - t_0 H_0^2) + \frac{q_0 t_0^2 H_0^2 + t_0^2 H_0^2}{t} + \frac{k}{a_0 e^{(H_0 - q_0 t_0 H_0^2 - t_0 H_0^2)(t - t_0)} \left(\frac{t}{t_0}\right)^{q_0 t_0 H_0^2 + t_0^2 H_0^2}} \right] - \frac{2\left[\frac{q_0 t_0^2 H_0^2 + t_0^2 H_0^2}{t^2} + \frac{K}{a_0^2 e^{2(H_0 - q_0 t_0 H_0^2 - t_0 H_0^2)(t - t_0)} \left(\frac{t}{t_0}\right)^{2(q_0 t_0^2 H_0^2 + t_0^2 H_0^2)}} \right]}{(\omega+1)} \tag{29}$$

Based on equation (28), we have determined the time-variation of $\frac{\dot{G}}{G}$. Figure (7) shows the variation of $\frac{\dot{G}}{G}$ as a function of cosmic time.

4. Study under a Time Dependent Equation of State (EoS) Parameter

In this section we have used the following empirical form (eqn. 30) which is capable of representing time-dependent EoS parameter (ω), as observed in graphical depictions based on the findings of various studies [38, 39].

$$\omega = C_1 a^{-n} + C_2 \text{ for } n > 0 . \tag{30}$$

Where a is the scale factor.

Now, considering $\omega = \omega_0$ for $a = a_0$, we get the following expression for C_2 as a function of C_1.

$$C_2 = -\frac{C_1}{a_0^n} + \omega_0 \tag{31}$$

Using equation (31) in equation (30), we get,

$$\omega = C_1 a^{-n} - \frac{C_1}{a_0^n} + \omega_0 \tag{32}$$

We can see from the equation (30),

$$\lim_{n\to\infty} \omega = C_2 \tag{33}$$

Equation (33) shows that the EoS parameter behaves as a time independent quantity for a large value of the parameter n.

We have found that, to get $G = G_0$ correctly at $t = t_0$, from equation (28), one needs to use $\omega = -0.722$. The present value of the gravitational constant is, $G_0 = 6.67 \times 10^{-11} Nm^2 Kg^{-2}$.

For $n = 1$, the time variation of the EoS parameter (ω) are shown in Figure (8) and Figure (9), for $C_1 < 0$ and $C_1 > 0$ respectively.

Now, as we are considering time-varying ω (equation 30), then from equation (10) and (11) we get,

$$\frac{d\rho}{dt} = -\frac{3\rho(1+\omega)\dot{a}}{a} \tag{34}$$

and,

$$\frac{d\rho}{dt} = -3\rho(1 + C_1 a^{-n} + C_2)\frac{\dot{a}}{a} \tag{35}$$

Solving equation (35) we can get the following expression for ρ.

$$\rho = C_3 e^{3C_1\frac{a^{-n}}{n} - 3(1+C_2)\ln a} \tag{36}$$

As $t \to t_0$,we have $a \to a_0$ and $\rho \to \rho_0$, then,

$$C_3 = \frac{\rho_0}{e^{3C_1\frac{a_0^{-n}}{n}-3(1+C_2)\ln a_0}} \tag{37}$$

Using equation (37) in equation (36) we get,

$$\rho = \rho_0 \frac{e^{3C_1\frac{a^{-n}}{n}-3(1+C_2)\ln a}}{e^{3C_1\frac{a_0^{-n}}{n}-3(1+C_2)\ln a_0}} \tag{38}$$

Equation (38) is the expression for ρ for the time-varying EoS parameter (ω). Here C_2 depends upon C_1, a_0 and ω_0 according to equation (31).

Now, keeping in mind the equation (16) and (17), we say $G = G(\omega, \rho)$ and $\Lambda = \Lambda(\omega, \rho)$. Thus, equations (32) and (38) need to be used to study the time-variations of the cosmological constant (Λ) and the gravitational constant (G), for the time-varyng EoS parameter (ω). It has been found that, to get $G = G_0$ (where $G_0 = 6.67 \times 10^{-11} Nm^2Kg^{-2}$) correctly at $t = t_0$, one needs to set $\omega_0 = -0.722$.

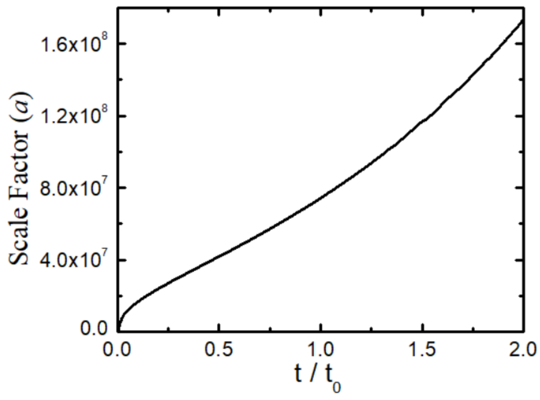

Figure 1: Plot of scale factor (a) versus time.

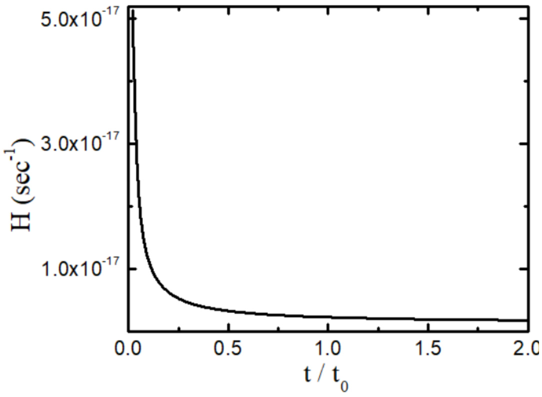

Figure 2: Plot of Hubble parameter (H) versus time.

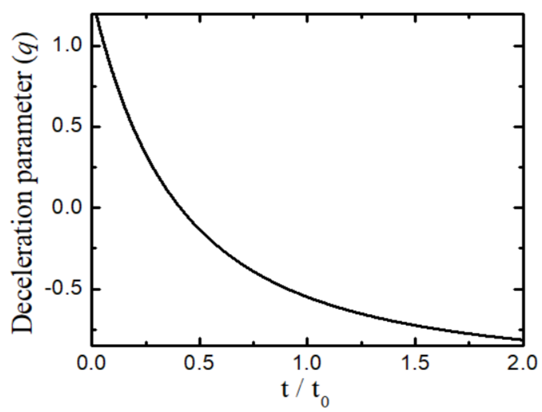

Figure 3: Plot of deceleration parameter (q) versus time.

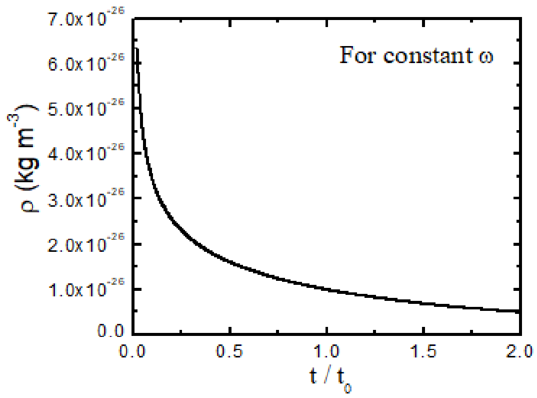

Figure 4: Plot of energy density of the universe (ρ) versus time.

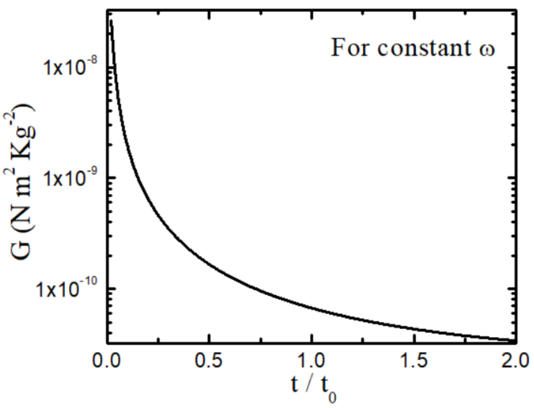

Figure 5: Plot of gravitational constant (G) versus time.

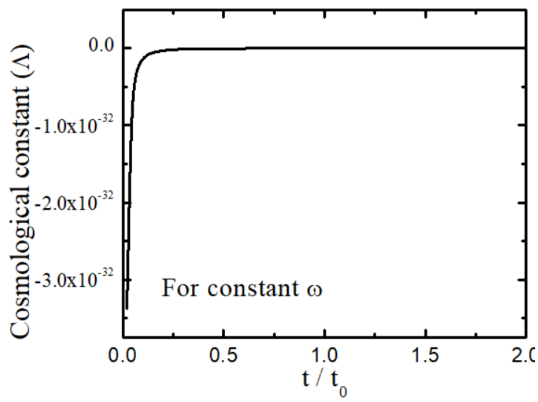

Figure 6: Plot of cosmological constant (Λ) versus time.

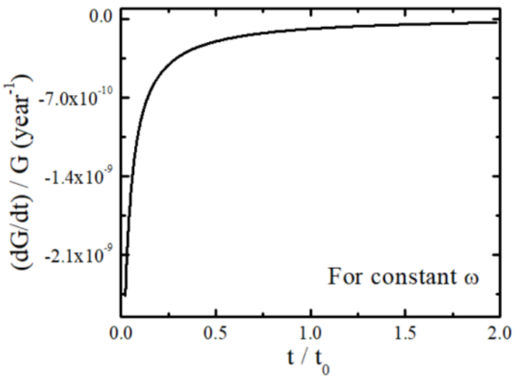

Figure 7: Plot of $\dot{G}/G$ versus time.

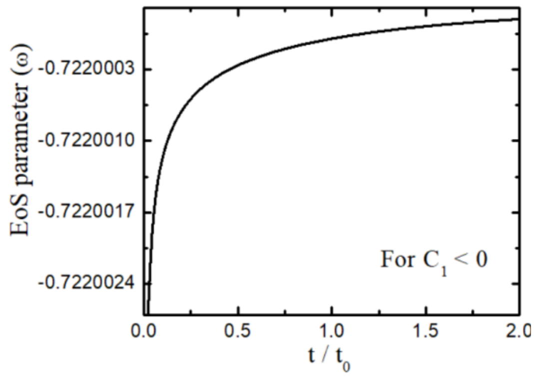

Figure 8: Plot of dynamical EoS parameter (ω) for $C_1 < 0$.

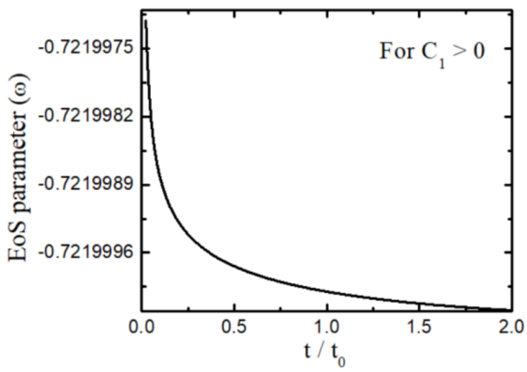

Figure 9: Plot of dynamical EoS parameter (ω) for $C_1 > 0$.

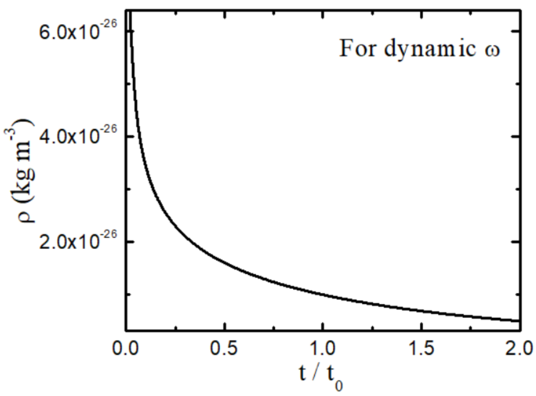

Figure 10: Plot of energy density (ρ) versus time, for a time-varying ω.

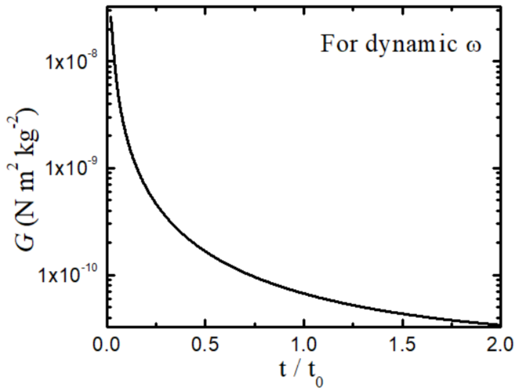

Figure 11: Plot of gravitational constant (G) versus time, for a time-varying ω.

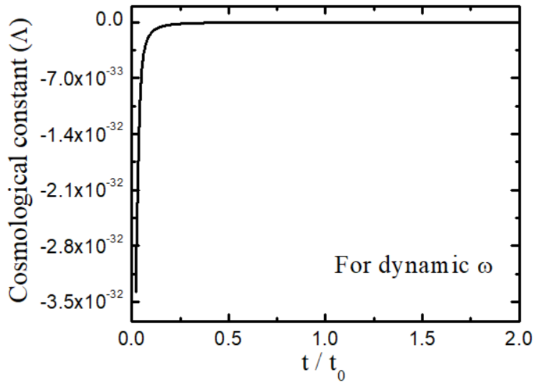

Figure 12: Plot of cosmological constant (Λ) versus time, for a time-varying ω.

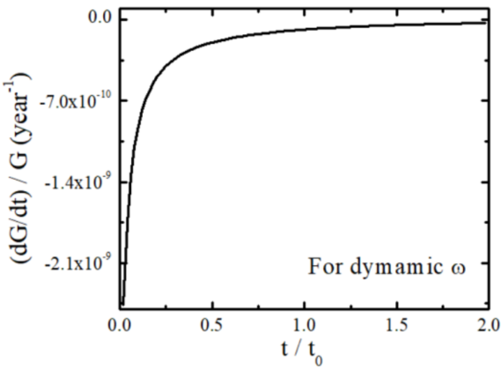

Figure 13: Plot of $\dot{G}/G$ versus time, for a time-varying ω.

5. Results and Discussion

In this study we have shown the time variation of some cosmological parameters such as Hubble parameter (H), deceleration parameter (q), energy density of the universe (ρ) along with the gravitational constant (G) and cosmological constant (Λ), showing their changes as functions of dimensionless cosmic time (t/t_0), for an empirical scale factor $a = Ae^{\alpha t}t^{\beta}$ (hybrid scale factor) where α, β are constants.

Figure 1 shows the time-variation of the scale factor (a), which increase with time for an expanding universe.

Figure 2 shows the cosmic time variation of the Hubble parameter (H), which is found to decrease with time monotonically.

Figure 3 shows the variation of deceleration parameter (q) as a function of time. It clearly shows the transition from decelerated expansion to accelerated expansion, by becoming negative from positive as time goes on. This variation also validates the choice of the empirical scale factor.

Figure 4 shows the variation of the energy density of the universe (ρ) for a constant EoS parameter (ω), which is, $\omega = -0.722$.

Figures 5 and 6 show the time variations of the gravitational constant (G) and cosmological constant (Λ) respectively, for $\omega = -0.722$. We find Λ to be increasing with time by being less negative from the early universe and saturating near zero at the present time. This rise in Λ may be due to the increase in dark energy which is held responsible for the accelerated expansion of the universe.

Figures 7 and 13 show the time variation of $\frac{\dot{G}}{G}$ for *constant* and *time-varying* ω, respectively. In these plots, at $\frac{t}{t_0} = 1$, one gets the present value of the same to be of the order of $10^{-11}\ year^{-1}$ which is consistent with the facts obtained from scientific literatures [1, 17].

Figures 8 and 9 show the time-variation of the equation of state parameter (ω), as per equation (32), for $C_1 < 0$ and $C_1 > 0$, respectively. In these two plots, ω is found to increase and decrease with time, respectively, becoming asymptotic to the value of C_2 as time goes on (in accordance with eqn. 30). We have taken $n = 1$ and $|C_1| = 25$ for these plots.

Based on the time-varying equation of state parameter (ω), we have plotted the time evolution of the energy density of the universe (ρ), gravitational constant (G) and cosmological constant (Λ), respectively, in Figures 10, 11 and 12. These results are consistent with the nature of variation of these parameters for constant EoS parameter (ω). Plots of ρ, G, Λ and $\dot{G}/G$ are found to be independent of whether C_1 is positive or negative, which means that these plots are independent of whether the evolution of ω is represented by Figure 8 or Figure 9.

The plot of $\dot{G}/G$ as a function of time in Figure 13, which depicts its evolution for the time-varying ω, is almost the same as the plot in Figure 7 which shows its behavior for a constant ω.

For the plots with constant ω, the value of ω is fixed at $\omega = -0.722$, because this value leads to the correct value of the gravitational constant (G) at the present time (i.e., $t = t_0$). For the plots of cosmological parameters for time-varying ω, the parameter ω_0 needs to be kept fixed at $\omega_0 = -0.722$ to obtain the correct value for G at the present time. The value of G at the present time is, $G_0 = 6.67 \times 10^{-11} Nm^2 Kg^{-2}$.

6. Conclusion

In this study, we have explored a cosmological model utilizing an empirically defined scale factor and examined the corresponding development of crucial dynamical parameters, including the Hubble parameter $H(t)$, deceleration parameter $q(t)$, energy density $\rho(t)$, as well as the time-dependent gravitational constant $G(t)$ and cosmological constant $\Lambda(t)$. We assessed the behavior of each parameter in relation to the dimensionless cosmic time $\left(\frac{t}{t_0}\right)$, with the current epoch defined as $\frac{t}{t_0} = 1$. The model effectively generates the accepted values of the cosmological quantities at the present time ($t = t_0$) for a certain choice of the constants associated with this formulation.

A significant aspect of this model is the behavior of the equation-of-state parameter $\omega(t)$, which converges to a stable asymptotic value in later times. Depending on the sign of the constant C_1 in the empirical formula for $\omega(t)$, it may either decrease steadily before stabilizing or rise initially and then settle at the same limiting value C_2. This stabilization of $\omega(t)$ is a characteristic feature observed in various dark-energy models [38, 39].

The behaviors of $G(t)$ and $\Lambda(t)$, predicted by our model, are also in agreement with established variable-constant cosmologies. Specifically, the monotonic or asymptotic trends of these quantities closely resemble those documented by some research groups [21, 22], who explored variable G and Λ through conservation laws. Similar patterns in evolution have been observed in certain studies [17, 18, 40], all of which underscore that a decreasing or dynamically stabilizing cosmological constant fits well within observational constraints. Furthermore, investigations by some researchers illustrate that a gravitational constant that changes slowly and approaches a constant value in the present epoch aligns with both theoretical perspectives and cosmological observations [28, 41]. We have plans to build more such cosmological models in future based on empirical scale factors and study the results obtained from them and examine the inter-model consistency.

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