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# Determination of Gravity Potential Differences and Height Systems for Survey Control on KNUST Campus, Ghana

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## Abstract

Traditional spirit leveling remains the primary method for establishing height control in engineering and geodetic work. However, its results are affected by the specific path taken and are not purely determined by physics. Heights obtained solely through geometric leveling depend on the chosen route and cannot be directly integrated into global or regional height systems based on gravity potential. This study measures gravity potential differences between survey control stations on the KNUST campus in Kumasi, Ghana, and calculates orthometric, dynamic, and normal heights in accordance with physical height theory. Precise leveling was carried out among 16 existing control stations using a digital level and invar bar-coded staff. At the midpoint of each segment, ground gravity measurements were taken with a LaCoste & Romberg G-944 gravimeter. These gravity data were corrected for drift and tides using Geosoft, and geopotential numbers were computed from the segment height differences and average gravity values. From these geopotential numbers, orthometric, dynamic, and normal heights were derived for all stations. The results confirm that leveled heights are not unique and demonstrate that geopotential numbers offer a path-independent method to define height on campus. This study presents a practical approach to improve gravity-based vertical control in a local network, providing a foundation for future integration into national and global height systems.

**Keywords:** gravity potential, geopotential number, orthometric height, dynamic height, normal height, precise levelling, gravimetry, KNUST.

## 1. Introduction

Height information supports engineering design, construction, drainage, flood risk assessment, and geodetic positioning. Traditionally, vertical control has been based on spirit leveling referenced to a local mean sea level datum. Though precise leveling can achieve millimeter-level internal accuracy, the resulting orthometric heights are not physically unique: repeating a leveling loop along different paths generally yields slightly different heights for the same point because the method ignores spatial variations in gravity along the leveling route.

In physical geodesy, the fundamental quantity is not height but gravity potential. The Earth's gravity field can be described by gravity potential W, which is the sum of gravitational and centrifugal

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potentials. Surfaces of constant potential—equipotential surfaces—are everywhere perpendicular to the direction of the plumb line. The geoid is one such equipotential surface that approximates global mean sea level (Moritz & Hofmann-Wellenhof, 2005). The difference in gravity potential between the geoid and a point P is expressed as the geopotential number $C\_P = W_0 - W\_P$, where $W_0$ is the potential at the geoid and $W\_P$ is the potential at point P. Geopotential numbers are path-independent and offer a natural way to define height. Various height systems, including orthometric, dynamic, and normal heights, can be derived from geopotential numbers by appropriate scaling with gravity or normal gravity (Jekeli, 200).

In Ghana, as in many countries, routine engineering surveys still rely heavily on classical leveling with limited integration of gravity information. On the KNUST campus in Kumasi, several control points exist with conventional heights, but their relationship to the gravity field has not been thoroughly investigated. Establishing geopotential numbers and gravity-consistent heights for these stations would enhance the physical significance of local height control and serve as a foundation for future geodetic integration.

This paper presents a campus-scale case study where gravity potential differences among existing survey stations are determined using precise leveling and relative gravimetry. The specific objectives are: (1) to measure precise height differences between selected control stations on KNUST campus; (2) to observe ground gravity at midpoints along the leveling routes and derive mean gravity values; (3) to compute gravity potential differences and geopotential numbers for the stations; and (4) to derive orthometric, dynamic, and normal heights and evaluate the limitations of traditional level heights.

## 2. Theoretical Background

For practical purposes, the geopotential number between the geoid and a point P can be calculated through line integrals of gravity along a leveling path: $C\_P = \int_0^H g\, dh \approx \Sigma\, g_i\, \Delta h_i$, where $g_i$ is the gravity at the i-th instrument station, $\Delta h_i$ is the corresponding height difference obtained from leveling, and the sum is taken along the plumb line between the geoid and P. In practice, $g_i$ is estimated using gravimeter readings at surface points along the leveling line (Lee et al, 2010). Because C_P is defined solely by potential differences, it does not depend on the specific leveling route, unlike traditional orthometric height differences. Geopotential numbers are typically expressed in geopotential units (g.p.u.), where 1 g.p.u. = 1 kgal m = 1000 gal m. With standard gravity near 0.98 kgal, the numerical values of C in g.p.u. are close to orthometric heights measured in meters.

From geopotential numbers, multiple height systems can be derived. Dynamic height is given by $H\_D = C / \bar{g}_0$, where $\bar{g}_0$ is a representative normal gravity value on a reference ellipsoid. Orthometric height is $H\_O = C / \bar{g}$, where $\bar{g}$ is the average actual gravity along the plumb line between the geoid and point P. Normal height is $H\_N = C / \bar{\gamma}$, where $\bar{\gamma}$ is the average normal gravity along the corresponding normal plumb line within a reference ellipsoid. The selection of a height system depends on the specific application and available data, but in all cases, the fundamental variable is the geopotential number.

## 3. Study Area and Data

The study was carried out on the main campus of Kwame Nkrumah University of Science and Technology (KNUST) in Kumasi, Ghana. The campus includes academic buildings, residential facilities, and road networks with moderate elevation. Existing survey control points established by the Department of Geomatic Engineering and partners are spread across the campus, including stations labeled KSB1, KSB2, KCP1, KCP2, SGA1–SGA13, and temporary turning points TP1 and TP6. These stations form a closed network with a total leveling distance of about 3.8 km and have traditional heights from previous surveys. The equipment used included a digital level with an Invar bar-coded staff for accurate leveling, a LaCoste & Romberg G-944 relative gravimeter; change plates and a tripod; a measuring tape, thermometer, and umbrella; and Geosoft software for gravity data reduction and analysis.

## 4. Methods

Network reconnaissance identified all existing control points suitable for precise leveling and gravity observations. Stations were inspected for stability, accessibility, and clear lines of sight. The final network consisted of 16 control points connected by two leveling laps, as shown in figures 1 and 3: Lap 1 (KSB2 – KCP1 – KCP2 – KSB1 – TP6 – TP1 – TP6 – KSB2) and Lap 2 (SGA1 – SGA2 – SGA3 – SGA5 – SGA6 – SGA7 – SGA10 – SGA11 – SGA12 – SGA13 – TP1 – TP6 – KSB1 – KCP2 – KCP1 – KSB2).

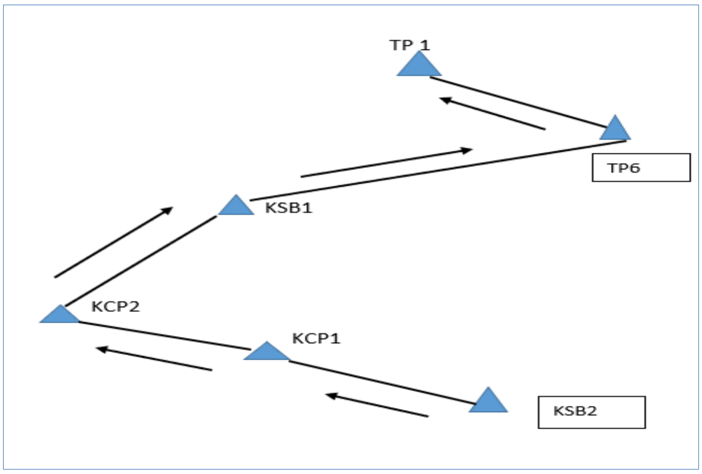


**Figure 1** Survey Diagram (Lap 1) and Direction of Levelling(1.7km)

**Figure 2** Lacoste and Rombergs Gravimeter

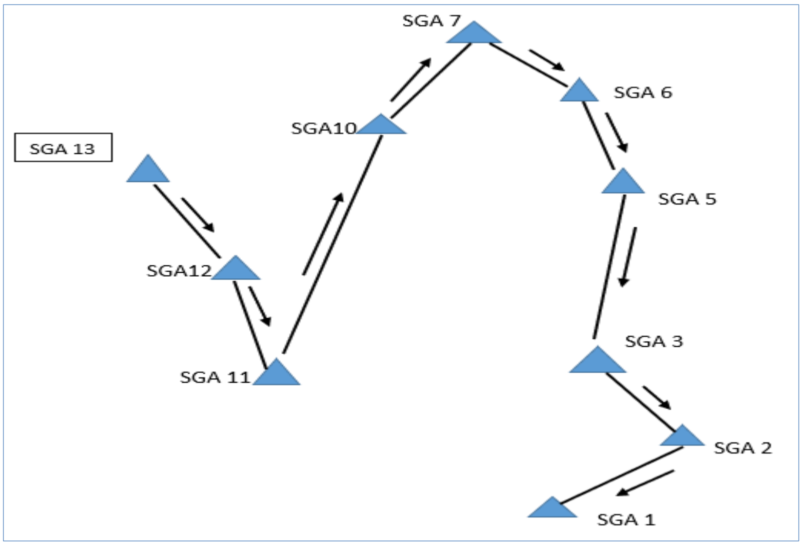


**Figure 3** Survey Diagram (Lap 2) and Direction of Levelling(2.1km)

Precise levelling followed first-order procedures with balanced backsight and foresight distances, short sight lengths, and two-way runs under different meteorological conditions. A two-peg test was performed to determine and correct residual collimation error. Height differences were determined using the rise-and-fall method and loop misclosures were checked against allowable tolerances.

Relative gravity measurements were made at the midpoints of each levelling segment using the L&R G-944 gravimeter (figure 3). The survey commenced and ended at a base station; base gravity was approximated using a theoretical normal gravity formula. Drift and tidal corrections were applied in Geosoft, and closure adjustments enforced consistency. For each levelling segment between stations A and B, the change in height Δh and the mean gravity g_m at the segment midpoint were known. Gravity potential differences were computed as ΔW_AB = − g_m Δh. Summation of segment contributions along a path from a reference station yielded geopotential numbers for all stations, from which orthometric, dynamic and normal heights were derived.

Table 1 and 2 show raw gravity readings and converted gravity readings in milligals respectivey. Calibration Table for G-944 was used to convert the raw gravity readings into gravity readings in milligals.

**Table 1** Raw Gravity Readings from Field

| | | | | |
|---|---|---|---|---|
| **Surveyed By: Moses Tangwam** | | | | |
| **Project: KNUST Ground Gravity - Ghana** | | | | |
| **DGPS: Sokia / Gravity Meter: L&R G944** | | | | |
| | | | | |
| **Station ID** | **Date YY/MM/DD** | **Time (Hrs Mins)** | **Gravity** | **Repeat** |
| SGA1_SGA2(Base) | 150320 | 1337 | 1673.68 | |

| | | | | |
|---|---|---|---|---|
| SGA2_SGA3 | 150320 | 1349 | 1674.73 | |
| SGA3_SGA5 | 150320 | 1408 | 1675.11 | |
| SGA5_SGA6 | 150320 | 1417 | 1674.20 | |
| SGA6_SGA7 | 150320 | 1426 | 1673.95 | |
| SGA7_SGA10 | 150320 | 1437 | 1672.48 | |
| SGA10_SGA11 | 150320 | 1444 | 1670.00 | |
| SGA11_SGA12 | 150320 | 1457 | 1670.29 | |
| SGA12_SGA13 | 150320 | 1504 | 1669.78 | |
| SGA1_SGA2 | 150320 | 1516 | 1673.84 | Yes |
| TP1_TP6 | 150320 | 1525 | 1671.37 | |
| TP6_KSB1 | 150320 | 1534 | 1673.69 | |
| KSB1_KCP2 | 150320 | 1542 | 1674.42 | |
| KCP2_KCP1 | 150320 | 1549 | 1674.11 | |
| KCP1_KSB2 | 150320 | 1557 | 1673.20 | |
| TP1_TP6 | 150320 | 1608 | 1671.41 | Yes |
| SGA1_SGA2 | 150320 | 1616 | 1673.90 | Yes |

**Table 2** Raw Gravity Readings in Milligals

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Surveyed By: Moses Tangwam** | | | | | | | | | |
| **Project: KNUST Ground Gravity – Ghana** | | | | | | | | | |
| **DGPS: Sokia / Gravity Meter: L&R G944** | | | | | | | | | |
| | | | | | | | | | **Gravity** |
| **Station ID** | **Date** | **Time** | **Gravity** | | | | | | **Readings** |
| | | | | | | | | | (mgal) |
| SGA1_SGA2(Base) | 150320 | 1337 | 1673.68 | **1600.00** | 73.68 | **1.03028** | 75.91 | 1649.52 | **1725.43** |
| SGA2_SGA3 | 150320 | 1349 | 1674.73 | **1600.00** | 74.73 | **1.03028** | 76.99 | 1649.52 | **1726.51** |
| SGA3_SGA5 | 150320 | 1408 | 1675.11 | **1600.00** | 75.11 | **1.03028** | 77.38 | 1649.52 | **1726.90** |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| SGA5_SGA6 | 150320 | 1417 | 1674.20 | **1600.00** | 74.20 | **1.03028** | 76.45 | 1649.52 | **1725.97** |
| SGA6_SGA7 | 150320 | 1426 | 1673.95 | **1600.00** | 73.95 | **1.03028** | 76.19 | 1649.52 | **1725.71** |
| SGA7_SGA10 | 150320 | 1437 | 1672.48 | **1600.00** | 72.48 | **1.03028** | 74.67 | 1649.52 | **1724.19** |
| SGA10_SGA11 | 150320 | 1444 | 1670.00 | **1600.00** | 70.00 | **1.03028** | 72.12 | 1649.52 | **1721.64** |
| SGA11_SGA12 | 150320 | 1457 | 1670.29 | **1600.00** | 70.29 | **1.03028** | 72.42 | 1649.52 | **1721.94** |
| SGA12_SGA13 | 150320 | 1504 | 1669.78 | **1600.00** | 69.78 | **1.03028** | 71.89 | 1649.52 | **1721.41** |
| SGA1_SGA2 | 150320 | 1516 | 1673.84 | **1600.00** | 73.84 | **1.03028** | 76.08 | 1649.52 | **1725.60** |
| TP1_TP6 | 150320 | 1525 | 1671.37 | **1600.00** | 71.37 | **1.03028** | 73.53 | 1649.52 | **1723.05** |
| TP6_KSB1 | 150320 | 1534 | 1673.69 | **1600.00** | 73.69 | **1.03028** | 75.92 | 1649.52 | **1725.44** |
| KSB1_KCP2 | 150320 | 1542 | 1674.42 | **1600.00** | 74.42 | **1.03028** | 76.67 | 1649.52 | **1726.19** |
| KCP2_KCP1 | 150320 | 1549 | 1674.11 | **1600.00** | 74.11 | **1.03028** | 76.35 | 1649.52 | **1725.87** |
| KCP1_KSB2 | 150320 | 1557 | 1673.20 | **1600.00** | 73.20 | **1.03028** | 75.42 | 1649.52 | **1724.94** |
| TP1_TP6 | 150320 | 1608 | 1671.41 | **1600.00** | 71.41 | **1.03028** | 73.57 | 1649.52 | **1723.09** |
| SGA1_SGA2 | 150320 | 1616 | 1673.90 | **1600.00** | 73.90 | **1.03028** | 76.14 | 1649.52 | **1725.66** |

After converting the raw gravity readings into milligals, we then used Geosoft to process the data by applying the necessary corrections. Table 3 shows the processed gravity data.

**Table 3** Processed gravity Data

| **Station** | **Time** | **Reading** | **Longitude** | **Latitude** | **Gravity(mgal)** | **TideCorr** | **Closure** |
|---|---|---|---|---|---|---|---|
| SGA1_SGA2(Base) | 13:37:00 | 1725.43 | 1.34.17.70 | 6.40.30.60 | 978102.174 | 0.1687 | 0 |
| SGA2_SGA3 | 13:49:00 | 1726.51 | | | 978103.236 | 0.1563 | * |
| SGA3_SGA5 | 14:08:00 | 1726.9 | | | 978103.596 | 0.1346 | * |
| SGA5_SGA6 | 14:17:00 | 1725.97 | | | 978102.65 | 0.1235 | * |
| SGA6_SGA7 | 14:26:00 | 1725.71 | | | 978102.375 | 0.112 | * |
| SGA7_SGA10 | 14:37:00 | 1724.19 | | | 978100.835 | 0.0976 | * |
| SGA10_SGA11 | 14:44:00 | 1721.64 | | | 978098.273 | 0.0882 | * |
| SGA11_SGA12 | 14:57:00 | 1721.94 | | | 978098.549 | 0.0704 | * |
| SGA12_SGA13 | 15:04:00 | 1721.41 | | | 978098.006 | 0.0607 | * |
| SGA1_SGA2 | 15:16:00 | 1725.6 | 1.34.17.70 | 6.40.30.60 | 978102.174 | 0.0442 | 0.045 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| TP1_TP6 | 15:25:00 | 1723.05 | | | 978099.614 | 0.0318 | * |
| TP6_KSB1 | 15:34:00 | 1725.44 | | | 978101.995 | 0.0196 | * |
| KSB1_KCP2 | 15:42:00 | 1726.19 | | | 978102.736 | 0.0089 | * |
| KCP2_KCP1 | 15:49:00 | 1725.87 | | | 978102.409 | -0.0003 | * |
| KCP1_KSB2 | 15:57:00 | 1724.94 | | | 978101.471 | -0.0105 | * |
| TP1_TP6 | 16:08:00 | 1723.09 | | | 978099.611 | -0.0241 | * |
| SGA1_SGA2 | 16:16:00 | 1725.66 | 1.34.17.70 | 6.40.30.60 | 978102.174 | -0.0335 | -0.018 |

## 5. Results

Conventional height levels for the main control stations typically span around 25 meters vertically across the network, and leveling misclosures remain within acceptable limits, indicating good internal consistency. Processed gravity values at the leveling midpoints range from approximately 978098 to 978103 mGal, showing modest variation throughout the campus. The largest potential differences occur along the steep section between SGA7 and SGA10. Using SGA1 as the reference, geopotential numbers are propagated through the network as shown in table 4. Orthometric heights derived from these geopotential numbers slightly differ from heights obtained through purely leveled methods, especially around loops where leveling paths diverge. This illustrates the non-uniqueness of classical orthometric heights and the path-independent property of heights based on geopotential.

**Table 4** Gravity potential differences calculated from processed gravity values and levelled height differences.

| Line | ΔH (m) | Mid-gravity (mGal) | No. of instrument stations | ΔC (g.p.u.) |
|---|---|---|---|---|
| SGA1_SGA2 | -12.422 | 978102.174 | 6 | -12.150 |
| SGA2_SGA3 | 3.386 | 978103.236 | 3 | 3.312 |
| SGA3_SGA5 | 1.148 | 978103.596 | 6 | 1.123 |
| SGA5_SGA6 | -0.008 | 978102.650 | 3 | -0.008 |
| SGA6_SGA7 | 2.527 | 978102.375 | 2 | 2.472 |
| SGA7_SGA10 | 20.986 | 978100.835 | 14 | 20.526 |
| SGA10_SGA11 | -4.196 | 978098.273 | 3 | -4.104 |
| SGA11_SGA12 | 2.112 | 978098.549 | 1 | 2.066 |
| SGA12_SGA13 | -0.109 | 978098.006 | 1 | -0.107 |

| | | | | |
|---|---|---|---|---|
| TP1_TP6 | 1.044 | 978099.614 | 1 | 1.021 |
| TP6_KSB1 | -14.339 | 978101.995 | 15 | -14.025 |
| KSB1_KCP2 | 1.402 | 978102.736 | 1 | 1.371 |
| KCP2_KCP1 | 2.383 | 978102.409 | 1 | 2.331 |
| KCP1_KSB2 | 5.695 | 978101.471 | 4 | 5.570 |

### 5.1 Units and conversions

Gravity values measured by the LaCoste & Romberg G-944 gravimeter are recorded in "scale units", which were converted to gravity in Gal and subsequently to milligals (mGal). One Gal is defined as 0.01 m/s², so that 1 mGal = $10^{-5}$m/s². In this study, all processed gravity values are expressed in mGal.

Geopotential numbers and potential differences are expressed in geopotential units (g.p.u.). One g.p.u. is defined as 10 m²/s². For small height differences between two nearby points A and B, the geopotential difference is approximated by

$$\Delta C = \frac{\bar{g}\,\Delta H}{10},$$

where $\bar{g}$is the mean gravity along the segment (in m/s²) and $\Delta H$is the height difference in meters. Using mGal instead of m/s², this becomes

$$\Delta C\ [\text{g.p.u.}] = \bar{g}\ [\text{mGal}] \times \Delta H\ [\text{m}] \times 10^{-6}.$$

As a numerical example, consider the segment SGA1–SGA2, which has a height difference $\Delta H = -12.422$m and a mid-segment gravity of $\bar{g} = 978102.174$mGal. Converting gravity to m/s² gives

$$\bar{g} = 978102.174 \times 10^{-5} = 9.78102174\ \text{m/s}^2$$

The corresponding geopotential difference in g.p.u. is

$$\Delta C = \bar{g}\,\Delta H/10 = 9.78102174 \times (-12.422)/10 = -12.15\ \text{g.p.u.}$$

Using the mGal form of the equation gives the same result:

$$\Delta C = 978102.174 \times (-12.422) \times 10^{-6} = -12.15\ \text{g.p.u.}$$

All entries in Table 4 were calculated using this relationship.

### 5.2 Error and uncertainty analysis

Several sources of error affect the derived geopotential numbers and heights. The most important are levelling misclosure, gravimeter drift and tidal corrections, and gravimeter scale calibration.

**Levelling misclosure:** Each levelling loop was constrained such that the misclosure did not exceed the first-order tolerance of $\pm k\sqrt{L}$mm, where $L$is the loop length in kilometres. For the KNUST network (maximum loop length = 2.1 km), the observed misclosures were within ±20 mm. When distributed by least-squares adjustment, the resulting uncertainty in height difference between adjacent control points is on the order of 0.2 mm, which translates into less than 0.1 g.p.u. in the corresponding geopotential numbers.

**Gravimeter drift and tidal corrections:** The LaCoste & Romberg G-944 readings were adjusted for instrumental drift through repeated measurements at the base station and for solid Earth tides using standard tidal models. The residual uncertainty in the corrected gravity values is estimated at about ±0.02–0.05 mGal, based on repeat measurements. For a typical campus-scale height difference of 10–20 m, this amounts to an uncertainty in geopotential difference of less than ±0.01 g.p.u., or roughly ±1 mm in the derived orthometric height.

**Calibration uncertainty:** The gravimeter scale factor was checked against known gravity differences in the local calibration range. A conservative estimate of the calibration uncertainty is 0.05%. For the observed gravity variations, this contributes at most a few tenths of a milligal to the total error budget, corresponding to only a few millimetres in height.

Considering these sources of error, the total uncertainty in the derived orthometric, dynamic, and normal heights is about 1–2 cm when compared to the traditional leveled heights. On the campus scale discussed here, the differences between height systems shown in Table 5 are mainly due to actual gravity variations rather than measurement noise.

Table 5 summarizes the geopotential numbers and derived height systems for all stations in the KNUST gravity–levelling network. The classical spirit-levelled heights range approximately from 249 to 277 meters, and the corresponding geopotential numbers span from about 244 to 271 g.p.u. This variation reflects the modest relief across the campus but already indicates that potential differences are large enough to produce measurable distinctions between the various height definitions.efinitions.

By construction, the orthometric heights $H_o$coincide with the classical spirit-levelled heights at the centimetre level, so the differences $H_o - H_{\text{class}}$are essentially zero for all stations. In contrast, the dynamic heights $H_d$are systematically smaller than the classical heights by roughly 0.16–0.18 m across the network, while the normal heights $H_n$are systematically larger than the classical heights by approximately 0.48–0.54 m. These offsets are an order of magnitude greater than the estimated 1–2 cm uncertainty from levelling, gravimeter drift, and calibration, indicating that the discrepancies between height systems in Table 5 are controlled by real gravity field variations rather than measurement noise.

Overall, the final height results confirm that, even within the limited spatial extent of the KNUST campus, the choice of height system (orthometric, dynamic, or normal) yields differences of several decimeters relative to classical spirit-levelled heights. This highlights the practical importance of using gravity-consistent heights for precise engineering and geodetic applications, especially when integrating campus networks into larger regional or national reference frames.

**Table 5** Final height summary for KNUST control stations

| Station | Cp (g.p.u.) | Ho (m) | Hd (m) | Hn (m) | Classical height H_class (m) | Ho - H_class (m) | Hd - H_class (m) | Hn - H_class (m) |
|---|---|---|---|---|---|---|---|---|
| SGA1 | 256.142 | 261.369 | 261.204 | 261.876 | 261.369 | 0.000 | -0.165 | 0.507 |
| SGA2 | 243.968 | 248.947 | 248.790 | 249.430 | 248.947 | 0.000 | -0.157 | 0.483 |
| SGA3 | 247.286 | 252.333 | 252.173 | 252.823 | 252.333 | 0.000 | -0.160 | 0.490 |
| SGA5 | 248.411 | 253.481 | 253.321 | 253.973 | 253.481 | 0.000 | -0.160 | 0.492 |
| SGA6 | 248.404 | 253.473 | 253.313 | 253.965 | 253.473 | 0.000 | -0.160 | 0.492 |
| SGA7 | 250.880 | 256.000 | 255.838 | 256.497 | 256.000 | 0.000 | -0.162 | 0.497 |
| SGA10 | 271.446 | 276.986 | 276.811 | 277.523 | 276.986 | 0.000 | -0.175 | 0.537 |
| SGA11 | 267.334 | 272.790 | 272.618 | 273.319 | 272.790 | 0.000 | -0.172 | 0.529 |
| SGA12 | 269.404 | 274.902 | 274.728 | 275.435 | 274.902 | 0.000 | -0.174 | 0.533 |
| SGA13 | 269.297 | 274.793 | 274.619 | 275.326 | 274.793 | 0.000 | -0.174 | 0.533 |
| TP1 | 257.426 | 262.680 | 262.514 | 263.190 | 262.680 | 0.000 | -0.166 | 0.510 |
| TP6 | 258.450 | 263.724 | 263.557 | 264.236 | 263.724 | 0.000 | -0.167 | 0.512 |
| KSB2 | 253.688 | 258.865 | 258.701 | 259.367 | 258.865 | 0.000 | -0.164 | 0.502 |
| KSB1 | 244.397 | 249.385 | 249.227 | 249.869 | 249.385 | 0.000 | -0.158 | 0.484 |
| KCP1 | 248.107 | 253.170 | 253.010 | 253.661 | 253.170 | 0.000 | -0.160 | 0.491 |
| KCP2 | 245.771 | 250.787 | 250.628 | 251.274 | 250.787 | 0.000 | -0.159 | 0.487 |

The three height types were derived from the geopotential numbers, $C_p$:

- Orthometric height (approximate):

$$H_o = \frac{C_p}{\bar{g}_P}$$

where $\bar{g}_P$is mean gravity along the plumb line

- Dynamic height:

$$H_d = \frac{C_p}{\gamma_{45}}$$

where $\gamma_{45}$ is normal gravity at 45° latitude (≈ 9.8062 m/s²).

- Normal height (simple approximation):

$$H_n = \frac{C_p}{\bar{\gamma}_P}$$

where $\bar{\gamma}_P$ is mean normal gravity along the normal to the reference ellipsoid.

**6. Discussion**

The study shows that even across a campus-wide network, the difference between classical leveling heights and geopotential-based height systems can be measured. Since gravity variations across KNUST are small, differences are minor, but they remain important for geodetic consistency and connecting to larger-scale vertical datums. Calculating gravity potential differences depends on the accuracy of both leveling and gravity measurements. The leveling procedures used align with first-order precise leveling, and the gravimeter provided consistent values with acceptable precision. A limitation was the lack of an absolute gravity station on campus, which means absolute geopotential numbers cannot yet be directly linked to global datums. Nonetheless, relative potential differences and derived heights are reliable and suitable for local uses such as vertical control, GNSS–geoid integration, and teaching. The methodology can be easily applied to other local networks in Ghana. With additional absolute gravity data, GNSS coordinates, and a regional geoid, these networks could be integrated into a unified national height system.

**7. Conclusions**

This study has shown a practical method for measuring gravity potential differences and establishing gravity-consistent height systems for a local control network on the KNUST campus in Kumasi, Ghana. By combining precise first-order leveling with relative gravimetry using a LaCoste & Romberg G-944 gravimeter, we calculated geopotential numbers for 16 control stations and derived corresponding orthometric, dynamic, and normal heights. The comparison with conventional levelled heights confirms the well-known limitation of geometric leveling: heights obtained without considering spatial gravity variations are path-dependent and thus not strictly physical.

Although gravity variations across the campus are minor, they are enough to create measurable differences ranging from millimeters to a few centimeters between purely levelled heights and heights from geopotential numbers. This emphasizes the importance of including gravity data when establishing vertical control, even at the campus or city level, especially where networks need to connect to regional or national height systems. The results also show that the necessary observational and computational methods can be carried out using equipment and software standard in many geomatics labs and survey organizations.

The workflow presented here offers a model for densifying gravity-consistent vertical control in other local networks in Ghana and similar settings. Future work should focus on establishing an absolute gravity benchmark on campus, integrating GNSS data and regional geoid models, and expanding the network to connect with the national leveling and gravity frameworks. These developments would enable KNUST and similar networks to serve as reliable nodes in developing a unified vertical datum for Ghana and the broader West African region.

**References**

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