International Journal of Science and Technology (IJST)
ISSN: 3049-1118, Volume- 3, Issue- 4 (October - December 2026)

A Novel Approach to Modified Heinz-Quarter Mean Labeled NM-Polynomial and Topological Indices of Chemical Graphs

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Abstract:

NM-polynomial is an extension of the classical M-polynomial and is based on Neighborhood -related vertex parameters.It provides a compact representation of the structural characteristics of a graph and serves as an effective tool in chemical graph theory for predicting physical properties.By incorporating both graph labeling information and graph topology,it yields more informative molecular descriptors. A topological index is a numerical function that maps a graph to a real number and characterizes its structural features.Such indices are widely used in QSPR and QSAR studies to investigate the relationship between molecular structure and physicochemical properties.In this article,we introduce a modified Heinz-Quarter mean labeling and the corresponding modified Heinz-Quarter mean labeled NM-Polynomial based on neighborhood label sums.Furthermore,we compute several induced topological indices for selected chemical graphs.

Keywords: Heinz-Quarter Mean Labeling,Modified Heinz-Quarter Mean Labeling,Modified HQM labeled NM-Polynomial.

Subject Classification (MSC2020): 05C31

1.Introduction:

Chemical graph theory provides a mathematical framework for representing chemical structures by graphs,in which atoms are represented by vertices and chemical bonds are represented by edges.Such molecular graph representations enable the structural characteristics of chemical compounds to be investigated using graph-theoretical parameters.Among the important descriptors in chemical graph theory,topological indices are numerical quantities associated with molecular graphs and have been widely employed to investigate relationship between molecular structure and physicochemical properties.The concept of topological indices in chemical graph theory originated from Wiener's work,which established a relationship between the structural characteristics of paraffins and their boling points[1].since

*Corresponding Author Email: lekshminithikaa@gmail.com
Published: 17 October 2026
DOI: https://doi.org/10.70558/IJST.2026.v3.i4.241340
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then, numerous degree-based ,distance-based and neighborhood -based topological indices have been developed for the structural analysis of chemical compounds.

The direct computation of several topological indices for a graph can become complicated.To overcome this difficulty,polynomial-based approaches have been developed as compact mathematical tools for generating different topological indices.In particular,the M-polynomial introduced by Deutsch and klavzar provides a systematic framework for deriving several degree -based topological indices through mathematical operators[2].This polynomial based approach has subsequently been applied to various chemical structures,polymers,nano tubes,benzenoid systems and other molecular networks.

A further development in this direction is the NM-polynomial,which includes the neighborhood degree-sum information of vertices rather than their ordinary degrees.Mondal et al[3,4] developed the NM-Polynomial framework and demonstrated its applicability to crystallographic structure,including the face-centered cubic lattice and cuprous oxide structures, titanium compounds including TiO_2 nanotubes,TiF_2 crystallographic structures and derived several neighborhood degree sum based topological indices.

Verma et al [5] studied bismuth tri-iodide and obtained the NM-polynomials of its chain and sheet structures.The NM-polynomial method has also been employed in the study of polycyclic aromatic hydrocarbons,where neighborhood M-polynomials were computed for three classes of polycyclic aromatic structures[6].The M-polynomial and NM-polynomial frameworks have also been extended to polymers.Mohammed Yasin et al[7] investigated certain biopolymers using both M-polynomial and NM-polynomial approaches and derived several topological indices.Similarly,subsequent studies have considered polymer structures and demonstrated the usefulness of polynomial methods for obtaining topological descriptors[8].

Further applications of the NM-polynomial have appeared in studies of cycle-related graphs,nano structures and graph operations.Studies on cycle -related graphs have used the neighborhood M-polynomial to derive several neighborhood degree-sum based topological indices[9].The NM -polynomial approach has also been applied to graph operations and to nano structural networks such as two-dimensional lattices,nanotubes and nanotorus[10,11].In addition,polynomial-based topological descriptors have been used for the molecular structures of compounds considered in the treatment of COVID-19,demonstrating in the applicability of M-polynomials and NM-polynomial methods[12].Ranganath et al[13] investigated the M-polynomial and NM-polynomial of the linear chain of indane and derived several degree-based and neighborhood degree-based topological indices associated with its molecular structure.

These developments indicate that the M-polynomial and NM-polynomial are effective algebraic tools for obtaining several topological indices from molecular graphs.However,the existing NM-polynomial mainly indicates neighborhood degree-sum information and does not include a vertex labeling scheme in its construction.Latha and S.S.Sandhya et al[14] introduced the Heinz-Quarter Mean Labeling and studied its properties on various graph classes.Motivated by these developments,the present study introduces a Modified Heinz-Quarter Mean Labeling and incorporates it into the construction of a Modified Heinz-Quarter Mean Labeled NM-polynomial and significant topological indices such as Modified Heinz-

Quarter mean neighborhood first Zagreb index, Modified Heinz-Quarter mean neighborhood second Zagreb index, Modified Heinz-Quarter mean neighborhood forgotten index,Modified Heinz-Quarter mean neighborhood redefined third Zagreb index,Modified Heinz-Quarter mean neighborhood Randic index at $\alpha = \frac{1}{2}$ .In the proposed approach,the neighborhood structural information of the molecular graph is combined with the assigned vertex-labeling information to construct the corresponding labeled polynomial.From the resulting Modified Heinz-Quarter Mean labeled polynomial,several topological indices are derived.Therefore,the present work extends the existing NM-polynomial framework by incorporating labeling information into the neighborhood-based polynomial representation.The proposed approach provides an alternative methodology for investigating the structural characteristics of chemical graphs through polynomial-based topological descriptors. The proposed polynomial is constructed on the fully expanded molecular graph G,in which all Hydrogen atoms are included.

2.Preliminaries.

2.1. Heinz-Quarter Mean Labeling

A graph G = (V, E) having p vertices and q edges. Let f: V(G)→{1,2,…$q+1$}

be an injective function. For a vertex labeling f, the induced edge labeling f (e = uv) is defined by

f(e=uv)= $\left\lfloor \frac{\sqrt[4]{f(u)f(v)}\left(\sqrt{f(u)}+\sqrt{f(v)}\right)}{2} \right\rfloor$ or $\left\lfloor \frac{\sqrt[4]{f(u)f(v)}\left(\sqrt{f(u)}+\sqrt{f(v)}\right)}{2} \right\rfloor$.Then f is called Heinz-Quarter mean labeling.A graph which admits Heinz-Quarter mean labeling is called Heinz-Quarter mean labeled graph.

2.2. Modified Heinz-Quarter Mean Labeling:

A Modified Heinz-Quarter Mean Labeling is a Heinz-Quarter Mean Labeling satisfying the condition f $(v_i) = i\ for\ i = 1,2,3 \ldots \ldots q+1$.

2.3. Modified Heinz-Quarter Mean Labeled NM-Polynomial:

Let N$S_f(u)$ be the sum of all Modified Heinz vertex labels neighbor to vertex u.

$$NS_f(u) = \sum_{v \epsilon N(u)} f(v)$$

$$NM^{mHQM}(G; x,y) = \sum_{i \leq j} n_{i,j}(G)\ x^i\ y^j$$

Where $n_{i,j}$ is the number of edges connecting two vertices whose neighborhood label sums are exactly i and j.

2.4. Modified Heinz-Quarter Mean Neighborhood First Zagreb Index:

Modified Heinz-Quarter Mean Neighborhood First Zagreb Index of a graph G is defined as the sum of the Modified Heinz-Quarter Mean Neighborhood label sums of the end vertices overall edges of G.

$$M_1^{mHQM} = \sum_{uv \in E(G)} [NS_f(u) + NS_f(v)]$$

2.5. Modified Heinz-Quarter Mean Neighborhood Second Zagreb Index:

Modified Heinz-Quarter Mean Neighborhood Second Zagreb Index of a graph G is defined as the sum of the products of the Modified Heinz-Quarter Mean Neighborhood label sums of the end vertices overall edges of G.

$$M_2^{mHQM} = \sum_{uv \in E(G)} [NS_f(u) NS_f(v)]$$

2.6. Modified Heinz-Quarter Mean Neighborhood Forgotten Index:

Modified Heinz-Quarter Mean Neighborhood Forgotten Index of a graph G is defined as the sum of the squares of the Modified Heinz Neighborhood label sums of the end vertices overall edges of G.

$$F^{mHQM} = \sum_{uv \in E(G)} [(NS_f(u))^2 + (NS_f(v))^2]$$

2.7. Modified Heinz-Quarter Mean Neighborhood Redefined third Zagreb Index:

Modified Heinz-Quarter Mean Neighborhood Redefined third Zagreb Index of a graph G is defined as the sum of the product of the Modified Heinz Neighborhood label sums and their corresponding sums overall edges of G.

$$ReZG_3^{mHQM}(G) = \sum_{uv \in E(G)} (NS_f(u) NS_f(v)) [NS_f(u) + NS_f(v)]$$

2.8. Modified Heinz-Quarter Mean Neighborhood Randic Index ($\alpha = \frac{1}{2}$):

Modified Heinz-Quarter Mean Neighborhood Randic Index of a graph G for $\alpha = \frac{1}{2}$ is defined as the sum of the square roots of the products of the Modified Heinz Neighborhood label sums of the end vrtices overall edges of G.

$$NR_{\frac{1}{2}}^{mHQM}(G) = \sum_{uv \in E(G)} \sqrt{NS_f(u) NS_f(v)}$$

3. Calculation of MHQM Neighborhood topological indices:

By replacing the neighborhood degree terms with Modified Heinz-Quarter Mean Neighborhood label sums $NS_f(u)$ and $NS_f(v)$,the corresponding labeled topological indices are obtained as follows. Let $NS_f(u) = D_x$ and $NS_f(v) = D_y$.

Table 3.1. Formulas derived from MHQM NM-Polynomial

MHQM Neighborhood Topological Indices	Derivatives from NM(G)
1.Modified Heinz-Quarter Mean Neighborhood First Zagreb Index (M_1^{mHQM})	(D_x+ D_y) NM^{mHQM} (G; x,y)/ x = y =1
2.Modified Heinz-Quarter Mean Neighborhood Second Zagreb Index (M_2^{mHQM})	(D_x D_y) NM^{mHQM} (G;x,y)/ x = y =1

3.Modified Heinz-Quarter Mean Neighborhood Forgotten Index (F^{mHQM})	$(D_x^2 + D_y^2)$ NM^{mHQM} (G;x,y) / x = y =1
4.Modified Heinz-Quarter Mean Neighborhood Redefined third Zagreb Index $\left(ReZG_3^{mHQM}(G)\right)$	(D_x D_y) (D_x+ D_y)NM^{mHQM} (G;x,y)/ x = y =1
5.Modified Heinz-Quarter Mean Neighborhood Randic Index($\alpha = \frac{1}{2}$) ($NR_{\frac{1}{2}}^{mHQM}$ (G))	$(D_x^{\frac{1}{2}}$ $D_y^{\frac{1}{2}})$ NM^{mHQM} (G;x,y)/ x = y =1

4. Chemical structures of Acids:

This section details the chemical structures of acids namely Trifluoroacetic acid (CF_3COOH), Oxalic acid ($C_2H_2O_4$), Phosphoric acid (H_3PO_4), Sulfuric acid (H_2SO_4), Lactic acid ($C_3H_6O_3$). The chemical structure of these acids are shown in Figure 1 to Figure 5.

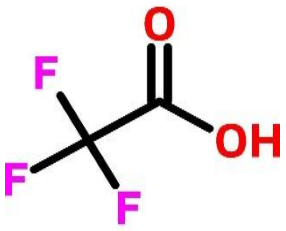


Figure 1.Trifluoroacetic acid

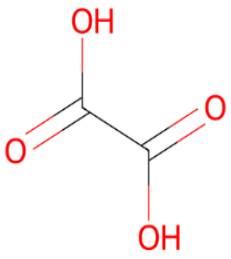


Figure 2.Oxalic acid

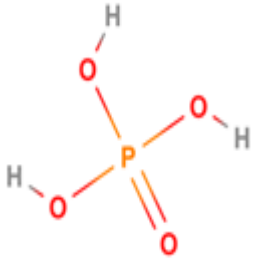


Figure 3.Phosphoric acid

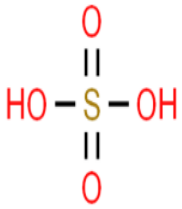


Figure 4.Sulfuric acid

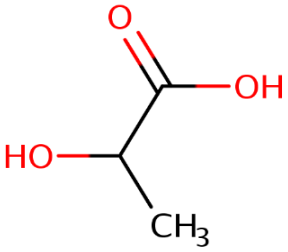


Figure 5.Lactic acid

Theorem: 4.1

Let G be a Modified Heinz-Quarter Mean Labeled structure of Trifluoroacetic acid then

(i) M-NM(G; x,y) = x^2y^4+ x^4y^{11}+ x^3y^{11}+ $x^{11}y^{24}$+3. x^5y^{24}

(ii) MHQM-Neighborhood First Zagreb Index = 157

(iii) MHQM-Neighborhood Second Zagreb Index = 709

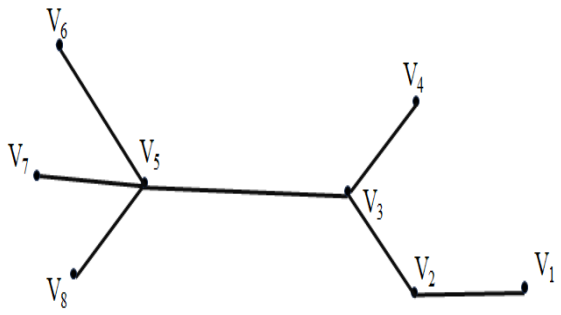
(iv) MHQM-Neighborhood Forgotten index = 2787

(v) MHQM-Redefined third Zagreb index = 20,850

(vi) MHQM-Neighborhood Randic index (R_α) $\alpha = \frac{1}{2} = 64.32$

Proof:

Let G be Molecular structure of Trifluoroacetic acid.It has 8 vertices and 7 edges and is shown in Figure 6 and Figure 7



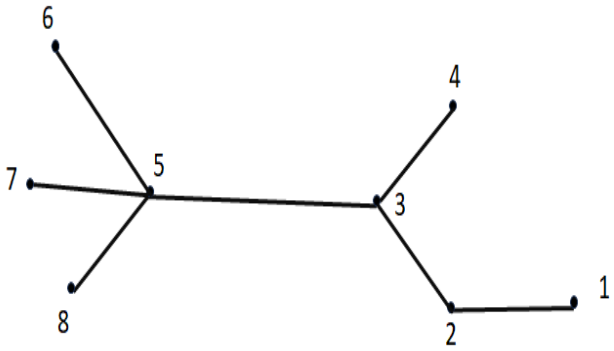


Figure:6. Molecular structure of Trifluoroacetic acid **Figure:7**. MHQM Labeling of G

The neighborhood sums are

Vertex	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8
Neighborhood sum	2	4	11	3	24	5	5	5

Then M-NM(G; x,y) = $x^2y^4 + x^4y^{11} + x^3y^{11} + x^{11}y^{24} + 3.\ x^5y^{24}$

(i)$(D_x + D_y) = 6\ x^2y^4 + 15x^4y^{11} + 14x^3y^{11} + 35x^{11}y^{24} + 3.\ (29)\ x^5y^{24}$

$(D_x + D_y)/\ x = y = 1 \quad = 157$

ii) $(D_x\ D_y) = 8x^2y^4 + 44x^4y^{11} + 33x^3y^{11} + 264x^{11}y^{24} + 3(120)\ x^5y^{24}$

$(D_x\ D_y)/\ x = y = 1 \quad = 709$

(iii) $(D_x^2 + D_y^2) = 20x^2y^4 + 137x^4y^{11} + 130\ x^3y^{11} + 697x^{11}y^{24} + 3(601)\ x^5y^{24}$

$(D_x^2 + D_y^2)/\ x = y = 1 \quad = 2787$

(iv) $(D_x\ D_y)\ (D_x + D_y) = 48x^2y^4 + 660x^4y^{11} + 462\ x^3y^{11} + 9240x^{11}y^{24} + 10440\ x^5y^{24}$

$(D_x\ D_y)\ (D_x + D_y)/\ x = y = 1 \quad = 20850$

(v) $(D_x^{\frac{1}{2}}\ \ D_y^{\frac{1}{2}}) = \sqrt{8}x^2y^4 + \sqrt{44}x^4y^{11} + \sqrt{33}x^3y^{11} + \sqrt{264}\ x^{11}y^{24} + 3\sqrt{120}\ x^5y^{24}$

$(D_x^{\frac{1}{2}}\ \ D_y^{\frac{1}{2}})\ /\ x = y = 1 \quad = 64.32$

Hence the proof.

Theorem: 4.2

Let G be a Modified Heinz-Quarter Mean Labeled structure of Oxalic acid then

(i) M-NM(G; x,y) = $x^2y^4+ x^4y^{11}+ x^3y^{11}+ x^{11}y^{16}+ x^5y^{16}+ x^{13}y^{16}+ x^7y^{13}$

(ii) MHQM-Neighborhood First Zagreb Index = 132

(iii) MHQM-Neighborhood Second Zagreb Index = 640

(iv) MHQM-Neighborhood Forgotten index = 1588

(v) MHQM-Redefined third Zagreb index = 15454

(vi) MHQM-Neighborhood Randic index $(R_\alpha)\ \alpha = \frac{1}{2} = 61.38$

Proof:

Let G be Molecular structure of Oxalic acid.It has 8 vertices and 7 edges and is shown in Figure 8 and Figure 9.

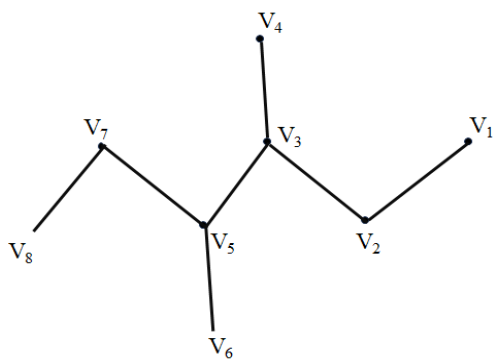


Figure 8. Molecular structure of Oxalic acid

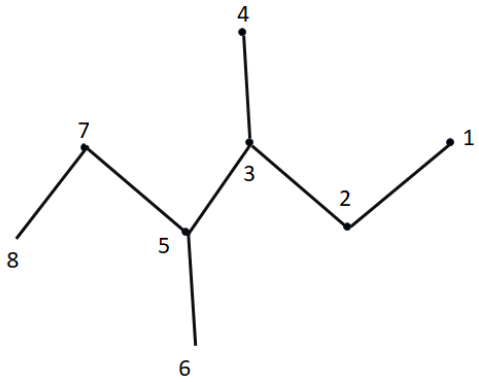


Figure 9.MHQM Labeling of G

The neighborhood sums are

Vertex	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8
Neighborhood sum	2	4	11	3	16	5	13	7

Then M-NM(G; x,y) = $x^2y^4+ x^4y^{11}+ x^3y^{11}+ x^{11}y^{16}+ x^5y^{16}+ x^{13}y^{16}+ x^7y^{13}$

(i)($D_x+ D_y$) = 6 $x^2y^4+ 15x^4y^{11}+14x^3y^{11} +27\ x^{11}y^{16}+ 21x^5y^{16}+29\ x^{13}y^{16}+ 20\ x^7y^{13}$

($D_x+ D_y$) / x = y =1 = 132

ii) ($D_x\ D_y$) = $8x^2y^4+ 44x^4y^{11}+33x^3y^{11}+ 176\ x^{11}y^{16}+80\ x^5y^{16}+ 208\ x^{13}y^{16}+ 91\ x^7y^{13}$

($D_x\ D_y$)/ x = y =1 = 640

(iii) ($D_x^2 + D_y^2$) = $20x^2y^4+ 137x^4y^{11}+ 130\ x^3y^{11}+ 377\ x^{11}y^{16}+ 281\ x^5y^{16}+ 425\ x^{13}y^{16}+ 218\ x^7y^{13}$

$(D_x^2 + D_y^2)$ / x = y =1 = 1588

(iv) (D_x D_y) (D_x+ D_y) = $48x^2y^4+ 660x^4y^{11}+ 462\ x^3y^{11} + 4752\ x^{11}y^{16} +1680\ x^5y^{16}+ 6032x^{13}y^{16}+ 1820\ x^7y^{13}$

(D_x D_y) (D_x+ D_y)/ x = y =1 = 15454

(v) $(D_x^{\frac{1}{2}}\ D_y^{\frac{1}{2}}) = \sqrt{8}x^2y^4+ \sqrt{44}x^4y^{11}+ \sqrt{33}x^3y^{11} +\sqrt{176}\ x^{11}y^{16}+ \sqrt{80}\ x^5y^{16} + \sqrt{208}\ x^{13}y^{16}+ \sqrt{91}\ x^7y^{13}$

$(D_x^{\frac{1}{2}}\ D_y^{\frac{1}{2}})$ / x = y =1 = 61.38

Hence the proof.

Theorem: 4.3

Let G be a Modified Heinz-Quarter Mean Labeled structure of Phosphoric acid then

(i) M-NM(G; x,y) = $x^2y^4+ x^4y^{18}+ x^3y^{18}+ x^9y^{18}+ x^5y^9+ x^{11}y^{18}+ x^7y^{11}$

(ii) MHQM-Neighborhood First Zagreb Index = 137

(iii) MHQM-Neighborhood Second Zagreb Index = 616

(iv) MHQM-Neighborhood Forgotten index = 1819

(v) MHQM-Redefined third Zagreb index = 14898

(vi) MHQM-Neighborhood Randic index (R_α) $\alpha = \frac{1}{2}$ = 60.94

Proof:

Let G be Molecular structure of Phosphoric acid.It has 8 vertices and 7 edges and is shown in Figure 10 and Figure 11.

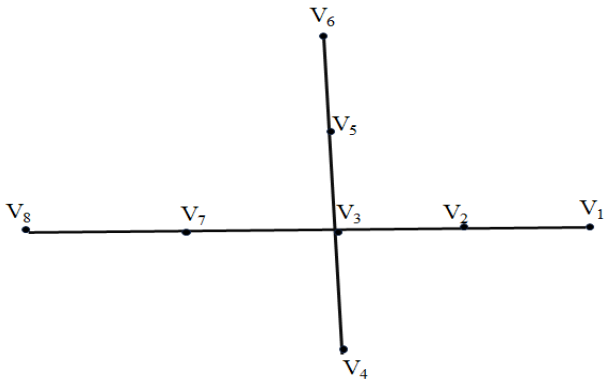


Figure 10. Molecular structure of Phosphoric acid

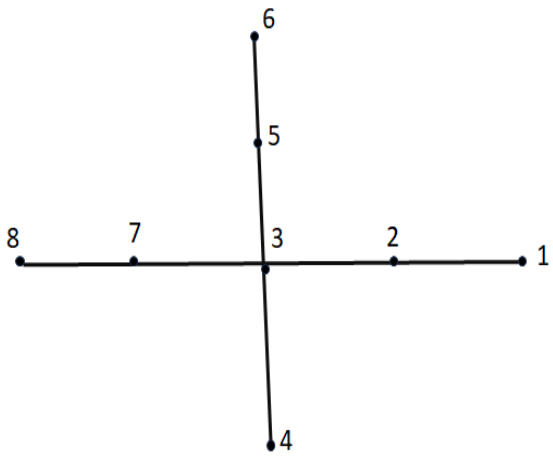


Figure 11. MHQM Labeling of G

The neighborhood sums are

Vertex	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8
Neighborhood sum	2	4	18	3	9	5	11	7

Then M-NM(G; x,y) = $x^2y^4+ x^4y^{18}+ x^3y^{18}+ x^9y^{18}+ x^5y^9+ x^{11}y^{18}+ x^7y^{11}$

(i)$(D_x+ D_y) = 6\ x^2y^4+ +22\ x^4y^{18}+21\ x^3y^{18}+ 27\ x^9y^{18}+ 14x^5y^9+ 29\ x^{11}y^{18}+ 18\ x^7y^{11}$

$(D_x+ D_y) / x = y =1 \quad = 137$

ii) $(D_x\ D_y) = 8x^2y^4+72\ x^4y^{18}+54\ x^3y^{18}+162\ x^9y^{18}+45\ x^5y^9+ 198\ x^{11}y^{18}+ 77x^7y^{11}$

$(D_x\ D_y)/ x = y =1 \quad = 616$

(iii) $(D_x^2 + D_y^2) = 20x^2y^4+ 340x^4y^{18}+ 333x^3y^{18}+ 405\ x^9y^{18}+ 106\ x^5y^9+ 445\ x^{11}y^{18}+ 170\ x^7y^{11}$

$(D_x^2 + D_y^2) / x = y =1 \quad = 1819$

(iv) $(D_x\ D_y) (D_x+ D_y) = 48x^2y^4+ 1584\ x^4y^{18}+ 1134\ x^3y^{18}+4374\ x^9y^{18}+630\ x^5y^9+5742\ x^{11}y^{18}+ 1386\ x^7y^{11}$

$(D_x\ D_y) (D_x+ D_y)/ x = y =1 \quad = 14898$

(v) $(D_x^{\frac{1}{2}}\ D_y^{\frac{1}{2}}) = \sqrt{8}x^2y^4+ \sqrt{72}x^4y^{18}+ \sqrt{54}x^3y^{18} +\sqrt{162}\ x^9y^{18}+ \sqrt{45}\ x^5y^9 + \sqrt{198}\ x^{11}y^{18}+ \sqrt{77}\ x^7y^{11}$

$(D_x^{\frac{1}{2}}\ D_y^{\frac{1}{2}}) / x = y =1 \quad = 60.94$

Hence the proof.

Theorem: 4.4

Let G be a Modified Heinz-Quarter Mean Labeled structure of Sulfuric acid then

(i) M-NM(G; x,y) = $x^2y^4+ x^4y^{17}+2\ x^3y^{17}+ x^{10}y^{17}+ x^6y^{10}$

(ii) MHQM-Neighborhood First Zagreb Index = 110

(iii) MHQM-Neighborhood Second Zagreb Index = 408

(iv) MHQM-Neighborhood Forgotten index = 1446

(v) MHQM-Redefined third Zagreb index ($ReZG_3$) = 9066

(vi) MHQM-Neighborhood Randic index (R_α) $\alpha = \frac{1}{2} = 46.14$

Proof:

Let G be Molecular structure of Sulfuric acid.It has 7 vertices and 6 edges and is shown in Figure 12 and Figure 13.

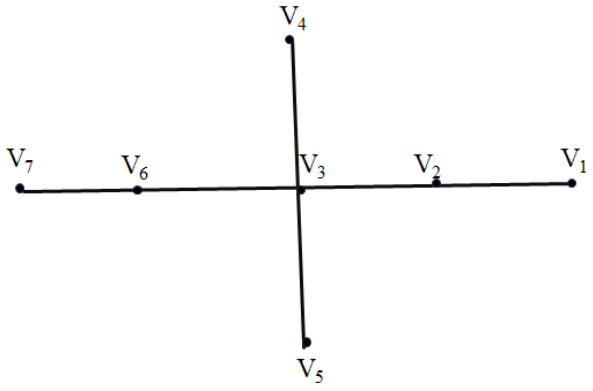


Figure 12. Molecular structure of Sulfuric acid

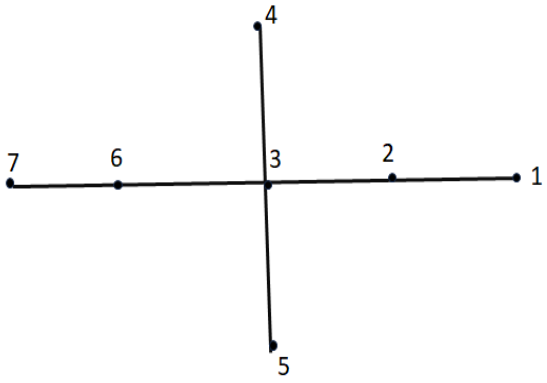


Figure 13. MHQM Labeling of G.

The neighborhood sums are

Vertex	V_1	V_2	V_3	V_4	V_5	V_6	V_7
Neighborhood sum	2	4	17	3	3	10	6

Then M-NM(G; x,y) = $x^2y^4 + x^4y^{17} + 2\,x^3y^{17} + x^{10}y^{17} + x^6y^{10}$

(i)$(D_x + D_y) = 6\,x^2y^4 + 21x^4y^{17} + 2(20)\,x^3y^{17} + 27x^{10}y^{17} + 16\,x^6y^{10}$

$(D_x + D_y) / x = y = 1 \quad = 110$

ii) $(D_x\,D_y) = 8x^2y^4 + 68\,x^4y^{17} + 2(51)\,x^3y^{17} + 170\,x^{10}y^{17} + 60\,x^6y^{10}$

$(D_x\,D_y) / x = y = 1 \quad = 408$

(iii) $(D_x^2 + D_y^2) = 20x^2y^4 + 305\,x^4y^{17} + 2(298)\,x^3y^{17} + 389\,x^{10}y^{17} + 136\,x^6y^{10}$

$(D_x^2 + D_y^2) / x = y = 1 \quad = 1446$

(iv) $(D_x\,D_y)(D_x + D_y) = 48x^2y^4 + 1428\,x^4y^{17} + 2\,(1020)\,x^3y^{17} + 4590\,x^{10}y^{17} + 960\,x^6y^{10}$

$(D_x\,D_y)(D_x + D_y) / x = y = 1 \quad = 9066$

(v) $(D_x^{\frac{1}{2}}\ D_y^{\frac{1}{2}}) = \sqrt{8}x^2y^4 + \sqrt{68}x^4y^{17} + 2\sqrt{51}x^3y^{17} + \sqrt{170}\,x^{10}y^{17} + \sqrt{60}\,x^6y^{10}$

$(D_x^{\frac{1}{2}}\ D_y^{\frac{1}{2}}) / x = y = 1 \quad = 46.14$

Hence the proof.

Theorem: 4.5

Let G be a Modified Heinz-Quarter Mean Labeled structure of Lactic acid then

(i) M-NM(G; x,y) = $x^2y^4+ x^4y^{11}+ x^3y^{11}+ x^{11}y^{25}+ x^5y^{25}+ x^{13}y^{25}+ x^7y^{13}+ x^{25}y^{38} + 3. X^9y^{38}$

(ii) MHQM-Neighborhood First Zagreb Index = 363

(iii) MHQM-Neighborhood Second Zagreb Index = 2877

(iv) MHQM-Neighborhood Forgotten index = 9339

(v) MHQM-Redefined third Zagreb index = 137062

(vi) MHQM-Neighborhood Randic index (R_α) $\alpha = \frac{1}{2}$ = 156.84

Proof:

Let G be Molecular structure of Lactic acid.It has12 vertices and 11 edges and is shown in Figure 14 and Figure 15.

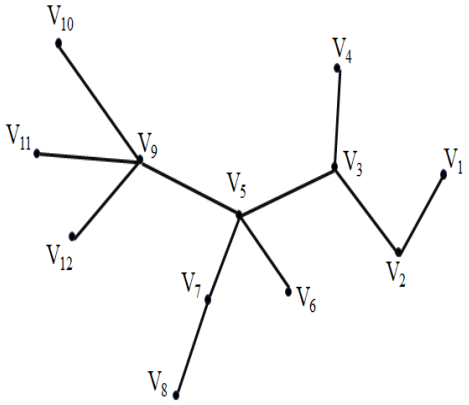


Figure 14. Molecular structure of Lactic acid

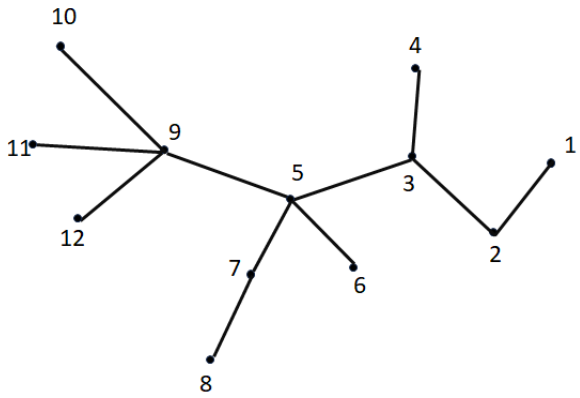


Figure 15. MHQM Labeling of G

The neighborhood sums are

Vertex	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	V_9	V_{10}	V_{11}	V_{12}
Neighborhood sum	2	4	11	3	25	5	13	7	38	9	9	9

Then M-NM(G; x,y) = $x^2y^4+ x^4y^{11}+ x^3y^{11}+ x^{11}y^{25}+ x^5y^{25}+ x^{13}y^{25}+ x^7y^{13}+ x^{25}y^{38} + 3. X^9y^{38}$

(i)($D_x+ D_y$) = 6 x^2y^4+ 15x^4y^{11}+14 x^3y^{11}+36 $x^{11}y^{25}$+ 30 x^5y^{25}+ 38 $x^{13}y^{25}$+20 x^7y^{13}+ 63 $x^{25}y^{38}$ + 3.

(47) x^9y^{38}

($D_x+ D_y$) / x = y =1 = 363

ii) $(D_x\ D_y) = 8x^2y^4 + 44\ x^4y^{11} + 33\ x^3y^{11} + 275\ x^{11}y^{25} + 125\ x^5y^{25} + 325\ x^{13}y^{25} + 91\ x^7y^{13} + 950\ x^{25}y^{38}$

$+\ 3.(342)\ x^9y^{38}$

$(D_x\ D_y)/\ x = y = 1 \quad = 2877$

(iii) $(D_x^2 + D_y^2) = 20x^2y^4 + 137\ x^4y^{11} + 130\ x^3y^{11} + 746\ x^{11}y^{25} + 650\ x^5y^{25} + 794\ x^{13}y^{25} + 218$

$x^7y^{13} + 2069\ x^{25}y^{38} + 3.(1525)x^9y^{38}$

$(D_x^2 + D_y^2)/\ x = y = 1 \quad = 9339$

(iv) $(D_x\ D_y)\ (D_x + D_y) = 48x^2y^4 + 660\ x^4y^{11} + 462\ x^3y^{11} + 9900\ x^{11}y^{25} + 3750\ x^5y^{25} + 12350\ x^{13}y^{25} +$

$1820\ x^7y^{13} + 59{,}850\ x^{25}y^{38} + 3.(16074)\ x^9y^{38}$

$(D_x\ D_y)\ (D_x + D_y)/\ x = y = 1 \quad = 137062$

(v) $(D_x^{\frac{1}{2}}\ D_y^{\frac{1}{2}}) = \sqrt{8}x^2y^4 + \sqrt{44}x^4y^{11} + \sqrt{33}x^3y^{11} + \sqrt{275}\ x^{11}y^{25} + \sqrt{125}\ x^5y^{25} + \sqrt{325}\ x^{13}y^{25} + \sqrt{91}$

$x^7y^{13} + \sqrt{950}\ x^{25}y^{38} + 3.\sqrt{342}\ x^9y^{38}$

$(D_x^{\frac{1}{2}}\ D_y^{\frac{1}{2}})\ /\ x = y = 1 \quad = 156.84$

Hence the proof.

CONCLUSION:

In this study,a modified Heinz-Quarter mean labeling was introduced and utilized to construct a modified Heinz-Quarter mean labeled Neighborhood M-polynomial for selected chemical acid graphs.Several labeled topological indices were derived from the proposed polynomial and analyzed to characterize the structural features of the considered chemical graphs.The results demonstrate that the proposed approach provides an effective framework for incorporating labeling information into neighborhood-based molecular descriptors.Further studies can focus on examining the relationships between the derived topological indices and various physicochemical properties of chemical compounds through regression analysis and QSPR studies.

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