

A Theoretical Study of Pressure Distribution of Magnetohydrodynamic (MHD) Flow Due to Exponentially Growing Source

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Abstract:

Navier-Stokes equations are limited under some approximation when the viscous forces are considerably greater than the inertia forces. When the velocity of a fluid flow is small, viscous forces are dominant. This kind flow is common when the Reynolds number is very small ($Re < 1$), under this condition the inertia terms may be omitted from the equation of motion of the fluid. This type of fluid motion does not occur in practical application but some time may occur in nature. This paper aimed to investigate the pressure distribution of a viscous incompressible fluid due an exponentially growing source between two parallel stationary disks in presence of uniform transverse magnetic field for small Reynolds number ($Re < 1$). Navier-Stokes equations are solved to evaluate pressure distribution of the fluid. Using computer programming, graphs are plotted to study the relative variation of pressure of the fluid with magnetic field parameter such as Hartmann number (M), reduced Reynolds number (Re^*) and growing factor of the source (n). Finally results that are found by analyzing graphs due to exponentially growing source are compared with similar work for exponentially decay source. This study investigates the pressure distribution of a viscous, incompressible fluid subjected to an exponentially growing source between two parallel stationary disks, in the presence of a uniform transverse magnetic field, for small Reynolds numbers. The results offer valuable contributions to the understanding of magnetohydrodynamic flows in low-Reynolds-number regimes, with potential applications in both theoretical and natural fluid flow phenomena.

Key words: Pressure distribution, viscous forces, Reynolds number, Hartmann number.

1.0 INTRODUCTION:

The fluid is regarded as a continuous medium. It follows that any small volume element in the fluid is considered so large that it contains a very great number of molecules. Moreover, for an ideal fluid we can define a fluid particle as the fluid contained with the physically infinitesimal volume i.e. very small compared with the volume of the body under consideration, but large compared with the distance between the molecules. An ideal fluid is supposed to have no viscosity, compressibility and surface tension etc. The branch of fluid mechanics which deals with ideal fluids is known as Hydrodynamics. When studies were made on different plasma phenomena, the equations of motion were found very similar to those which were used in

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studying hydrodynamic phenomena for many years. The study of plasma or hydrodynamics were carried out under the action of magnetic field using hydrodynamic equations and is now to describe the whole field of plasma studies is the study of microscopic interaction of electrically conducting liquids and gases with magnetic field. The subject can further be divided into two branches. Magneto-hydrodynamics and Magneto-gasdynamics. There are many problems in which the energy in the electric field is much smaller than that in the magnetic field. In these cases, we may express all the electromagnetic quantities in terms of magnetic field. As a result, we shall consider only the interaction between the magnetic field and the gasdynamic field. This analysis is well known “Magnetogasdynamics”. ^{[1][2]}

The interlocking between the mass motion and the electromagnetic field is the Magnetohydrodynamic (MHD) phenomena. The phenomena generated in the electrically conducting fluid due to the interaction of electromagnetic field with the velocity field are known as Magnetohydrodynamics. The motion of the conducting fluid across the magnetic field generates electric currents which change the magnetic field and the action of the magnetic field on these currents gives rise to mechanical forces which modify the flow. In Magnetohydrodynamics, the compressibility effect of the medium is negligible. Therefore MHD is the electromagnetic phenomenon in the flow of an incompressible electrically conducting fluid. ^{[2][3][4]}

There are many important applications, such as in the problems of controlled thermo-nuclear reactors for large scale power generation, thermonuclear fusion, of MHD in various fields both experimental as well as theoretical. It is not possible to predict the future developments of MHD. The data received from American and Russian satellites and space probes have already posed many new problems and lead to conclusions hardly conceivable a few years ago; also in the field of nuclear engineering the number of problems continues to multiply. It may be that we are still only on the threshold of this new subject. ^{[4][5][6]}

Laminar flow due to an exponential source between two parallel stationary infinite disks studied by Gourla and Mehta (1994). They found out solutions for the motion of the liquid in the form of an infinite series expanded in terms of deduced Reynolds number Re^* . Their solutions are found valid for small values of reduced Reynolds number. They analyzed the dependents of convective inertia over the viscous motion. The conclusion of their study was that the effects of nonlinear inertia is significant if the value of the decay factor (n) of the source of fluid is small, but the effect is less significant if the value of the decay factor is higher. But it is possible to extend the field of application to larger value of Reynolds number using successive approximation method. If we carry out more than a few step in the approximation, calculations are found complicated. ^{[6][7]}

We have discussed pressure distribution of a viscous incompressible fluid due an exponentially growing source between two parallel stationary disks in presence of uniform transverse magnetic field for small Reynolds number ($Re < 1$). The results are obtained due to simultaneous effect of inertia and magnetic field on radial velocity of the fluid. The relative variations of pressure distribution due to an exponentially growing source than exponentially decay source are discussed. ^{[8][9]}

2.0 THE PROBLEM:

When the velocity of a fluid is small, viscous forces are dominant. This kind of flow is common when the Reynolds number is very small. The fluid motion when Reynolds number is less than one is called creeping motion. The investigation of the paper is a modest attempt to obtain the flow pattern of a viscous incompressible fluid due to an exponentially growing source between two parallel stationary disks in presence of transverse magnetic field for small Reynolds number ($Re < 1$). To study the flow pattern, there is need to solve hydrodynamic equations to evaluate radial velocity due to simultaneous effect of inertia and magnetic field.

3.0 OBJECTIVES:

- 1) To study the pressure distribution of conducting incompressible and viscous fluid in presence of transverse magnetic field between two circular plates for exponentially growing source.
- 2) To develop mathematical idea for solving MHD problem with some assumptions.
- 3) To study pressure distribution for exponentially growing source between two circular plates in presence of transverse magnetic field.
- 4) To compare results, obtain for exponentially growing source with similar work for exponentially decay source.

Comparisons are made on following aspects-

- a) To study effects of reduced Reynolds number (Re^*) on pressure distribution for some different values of growing factor (n).
- b) To study effect of growing factor under uniform transverse magnetic field on pressure distribution.
- c) To study the effect on pressure distribution of the fluid for constant growing factor and Reynolds number with the increase of magnetic Hartmann number (H).

4.0 HYPOTHESIS:

1. Under the uniform transverse magnetic field the nature of variation of the pressure distribution of the Conducting fluid between two circular plates, with the increase of Reynolds number is not similar for exponentially increasing and exponentially decay source.
2. Under the uniform transverse magnetic field the variation of the pressure distribution is dependent on the growing factor of the source if we consider the fluid motion between two circular plates.
3. There is significant effect of reduced Reynolds number on pressure distribution for all values of growing factor for the exponentially increasing source of the fluid.
4. For exponentially increasing source of conducting fluid between two circular plates in presence of uniform transverse magnetic field, nonlinear effect of convective inertia on pressure distribution become significant for any value of growing factor.
5. For constant growing factor and Reynolds number, the pressure distribution of the fluid from an exponentially increasing source under transverse magnetic field increases with the increase of magnetic Hartmann numbers unlike exponentially decay source.

5.0 METHODOLOGY:

Step 1: The topic Magnetohydrodynamics is chosen because of interest.

Step 2: After a thorough study of the topic the problem is selected due to importance in modern science and technology.

Step 3 : The problem of the study is clearly stated.

Step 4: An extensive literature survey from books, existing thesis and journals are undertaken.

Step 5: The problem is solved with the help of MHD equations under certain approximation.

Step 6: To obtain the result various graphs are plotted using computer programming.

Step 7: Analyzing graphs, results and conclusions are drawn. [4][7] [8]

6.0 FORMULATION OF THE PROBLEM:

Considered an unsteady axially symmetric flow of a viscous incompressible fluid between two parallel stationary infinite disks. A cylindrical polar co-ordinate system is considered such that the disks are at $Z = \pm h$. By the term source it means that a point from which fluid is imagined to flow out uniformly in all direction surrounding the point. The total flux outwards across a small closed surface surrounding the point (Q) called the strength of the source. The line source of the fluid is situated on the Z- axis at $r=0$, whose strength varies according to following equation-

$$Q(t) = Q_0 e^{-nt} \text{-----(1)}$$

A transverse magnetic field B_0 is imposed perpendicular to the disks. u and v are the velocity components along radial and Z directions respectively. The uniform magnetic field of strength B_0 applied along the Z-axis. In order to derive the governing equations of the problem the following assumptions are made -

- (i) The fluid is finitely conducting and the viscous dissipation and Joules heat are neglected.
- (ii) Hall effect and polarization effect are neglected.
- (iii) The circular plates are at a constant temperature are stationary and the fluid viscosity varies with temperature only.
- (iv) The perturbation technique which is used for small values of the magnetic parameter (m) depending upon the degrees of magnetic field interaction, shows the effect due to the magnetic field by the second order term.
- (v) The value of magnetic Reynolds number is small.

7.0 GOVERNING EQUATIONS AND SOLUTIONS:

In MHD, the fluid is electrically conducting, therefore, the magnetic fluid affects mass motion, not because of its mere pressure but only by virtue of electric current, with velocity 'u' (along radial) in presence of a magnetic field B₀, then the force per unit volume can be written as -

$$\mathbf{f} = \rho_e \mathbf{E} + \mathbf{j} \times \mathbf{B}_0 \text{ -----(2)} \quad | \rho_e = \text{charge density}$$

Imposing axial symmetry, and using curl E=0, the magnetic body force,

$$\mathbf{f} = \mathbf{j} \times \mathbf{B}_0 \text{ ----- (3)}$$

$$\text{where } \mathbf{j} = \nabla \times \mathbf{B}_0 \text{ ----- (4)} \quad | \nabla = \text{current density}$$

$$\text{This gives } j_r = 0, j_\theta = -\frac{1}{r} \frac{\partial}{\partial z} (B_z u), j_z = \frac{1}{r} \frac{\partial}{\partial r} (B_z u)$$

Hence from equation (3), we get -

$$f_r = -\frac{1}{r} \frac{\partial}{\partial z} (B_z^2 u) = \frac{1}{r} \frac{\partial}{\partial z} (B_z^2 u), f_\theta = 0, f_z = 0$$

The equation of continuity and Navier-stokes equation in cylindrical polar coordinates r, θ and z for this problem are –

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial u}{\partial z} = 0 \text{ -----(5)}$$

$$\rho \left[\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + v \frac{\partial u}{\partial \theta} \right] = - \frac{\partial p}{\partial r} + \mu \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \left(\frac{\partial u}{\partial r} \right) - \frac{u}{r^2} + \frac{\partial^2 u}{\partial z^2} - \frac{1}{r} \frac{\partial}{\partial z} (B_z^2 u) \text{ -----(6)}$$

$$\rho \left[\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial r} + v \frac{\partial v}{\partial \theta} \right] = - \frac{\partial p}{\partial \theta} + \mu \frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \left(\frac{\partial v}{\partial r} \right) - \frac{v}{r^2} - \frac{\partial^2 v}{\partial z^2} \text{ -----(7)}$$

The boundary conditions of the problem are-

$$u=0, v=0 \text{ at } z=\pm h \text{ -----(8)}$$

$$\text{Total flux} = \int_{-h}^{+h} 2\pi r u \, dz = Q_0 e^{nt} \text{ -----(9)}$$

Introducing the following non-dimensional quantities-

$$r' = r/h, z' = z/n, v' = hv/v, u' = hu/\pi \text{ ----- (10) here } \nu = \text{kinematic shear viscosity}$$

$$P' = (1/P)(h/\nu)^2, n' = (h^2/\nu)n, t' = (\nu/h^2)t$$

Substituting equation (10) in equation (5-7) and then removing the primes, we get

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial u}{\partial z} = 0 \text{ -----(11)}$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + v \frac{\partial u}{\partial z} - \frac{\partial^2 u}{\partial r^2} - \frac{1}{r} \left(\frac{\partial u}{\partial r} \right) + \frac{u}{r^2} - \frac{\partial^2 u}{\partial z^2} + M^2 u = 0 \text{ -----(12)}$$

$$\text{and } \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial r} + v \frac{\partial v}{\partial z} - \frac{\partial^2 v}{\partial z^2} + \frac{1}{r} \left(\frac{\partial v}{\partial r} \right) - \frac{v}{r^2} - \frac{\partial^2 v}{\partial z^2} + \frac{\partial p}{\partial z} = 0 \text{ -----(13)}$$

Where $h^2 B_0^2 \nu / \rho \pi = M$ is the magnetic field parameter known as Hartmann number. Using equation (10) in (8) and (9), the corresponding boundary conditions are-

$$u=0 \text{ and } v=0 \text{ at } z = \pm 1 \text{ -----(14)}$$

$$\int_{-1}^{+1} u dz = 2\pi R_e / r e^{nt} \text{-----} (15)$$

Where $Q_0/4\pi\nu h = R_e$

R_e is the Reynolds number of the fluid also the controlling factor of the source. In order to solve the Equations (11-13) under boundary conditions (14) and (15) we consider following expansions:

$$u = R_e / r [f_0(z,t) + Re^* [f_1(z,t)] + (Re^*)^2 [f_2(z,t) + ...] \text{-----} (16)$$

$$v = 2(R_e^*)^2 [f_1(z,t) + 4(R_e^*)^3 [f_2(z,t)] + ...] \text{-----} (17)$$

$$p = [k(z,t) + Re[k_0(z,t) \log(r)] + (Re^*) [k_1(z,t) + ...] \text{-----} (18)$$

Where $Re^* = Re/2$, reduced Reynolds number. The primes denote partial differentiation with respect to z only.

Using the expansions (16-18), corresponding boundary conditions in terms of $f(z,t)$ and $f'(z,t)$ are given as :-

$$f_i(\pm 1, t) = 0 \quad \text{where } i=1,2 \text{-----} (19)$$

$$f'_i(\pm 1, t) = 0 \quad \text{where } i=0,1,2,3 \text{-----} (20)$$

$$\text{and } f_0(1,t) - f_0(-1,t) = 2e^{nt} \text{-----} (21)$$

Far streamline flow, we can consider -

$$f_0(1,t) = e^{nt} \text{-----} (22)$$

$$f_0(-1,t) = -e^{nt} \text{-----} (23)$$

So that they satisfy (21)

Using the expansions (16) and (17) and equating the co-efficient of like powers of r , we have from equations (12) and (13)

$$\left(\frac{\partial^3 f_0}{\partial z^3}\right) - \left(\frac{\partial^2 f_0}{\partial z^2} \frac{\partial}{\partial t}\right) + M^2 \left(\frac{\partial f_0}{\partial x}\right) = k_0(z,t) \text{-----} (24)$$

$$\frac{\partial k_0}{\partial z} = 0 \text{-----} (25)$$

$$\left(\frac{\partial^3 f_1}{\partial z^3}\right) - \left(\frac{\partial^2 f_1}{\partial z^2} \frac{\partial}{\partial t}\right) - M^2 \left(\frac{\partial f_1}{\partial x}\right) = 2k_1 \left[\left(\frac{\partial f_0}{\partial x}\right)^2 \right] \text{-----} (26)$$

$$\frac{\partial k_1}{\partial z} = 0 \text{-----} (27)$$

$$\text{Also, } \frac{\partial k}{\partial z} = 0 \text{-----} (28)$$

This implies that $K(z,t) = k(t)$

The relations (22) and (23) suggest that -

$$f_i(z,t) = C_i(z) e^{(i+1)nt} \text{-----} (29)$$

$$\text{And } K_i = \pi e^{(i+1)nt}$$

$$\text{Where } i = 0, 1 \text{ -----(30)}$$

Substituting (28) and (29), equation (22) and (23) becomes

$$C_0'''(z) + a C_0'(z) = P_0 \text{ ----- (31)}$$

$$\text{and } C_1'''(z) + b C_1'(z) + [2p_1 + \{f_0'(z)\}^2] \text{ -----(32)}$$

$$\text{Where } a = M_2 - n \text{ ----- (33)}$$

$$\text{and } b = M_2 - 2n \text{ ----- (34)}$$

The corresponding boundary conditions of the equation (31) and (32) from (22) and (23) are given as -

$$C_0(\pm 1) = \pm 1, C_0'(\pm 1) = 0 \text{ ----- (35)}$$

$$C_1(\pm 1) = 0, C_1'(\pm 1) = 0 \text{ ----- (36)}$$

and the solutions of equations, (25) to (27), subject to the boundary conditions (35) and (36), are obtained as –

$$f_0(z, t) = \{ z a \cos \sqrt{\alpha/A} - \sin \sqrt{\alpha z/A} \} e^{nt} \text{ ----- (37)}$$

$$K_0(t) = \{ a^{3/2} \cos \sqrt{\alpha/A} \} e^{nt} \text{ ----- (38)}$$

$$\begin{aligned} f_1(z, t) = & G \sin \sqrt{\beta z/A} - (2p_1/\beta)Z - (a/2A^2 \beta) (2\cos^2 \sqrt{\alpha+1})Z \\ & + \sin^2 \sqrt{\alpha z}/(4\sqrt{\alpha}(\beta-4\alpha) - (2\cos \sqrt{\alpha} \sqrt{\beta} \sin \sqrt{\alpha z} \sqrt{\alpha(\beta-\alpha)})) e^{2nt} \\ k_1(t) = & [\beta/2A^2] (1/B) \{ 3\sqrt{\alpha-\beta} \sqrt{\alpha} \sqrt{\beta} \sin 2\sqrt{\alpha} \}/4(\beta-4\alpha)(\beta-\alpha) \\ & - (\cos 2\alpha/2(\beta-4\alpha) - 2\cos^2 \sqrt{\alpha}/\beta - \alpha) \alpha \sin \sqrt{\beta} \\ & - \alpha [4A^2 2\cos^2 \alpha + 1] e^{2nt} \text{ ----- (39)} \end{aligned}$$

$$\text{Where } B = \sin \sqrt{\beta} - \sqrt{\beta} \cos \sqrt{\beta}, A = \sqrt{\alpha} \cos \sqrt{\alpha} - \sin \sqrt{\alpha} \text{ ----- (40)}$$

$$\begin{aligned} G = & 1/\sin \sqrt{\beta} \{ \alpha/2 \beta A^2 (2\cos^2 \alpha \sqrt{\alpha+1}) + \sqrt{\alpha}/A^2 (3(5\alpha-3) \\ & \sin 2\sqrt{\alpha}/(4(\beta-4\alpha)(\beta-\alpha)) + 2p_1/\beta \} \text{ ----- (41)} \end{aligned}$$

Differentiating equations (37) and (39), we get –

$$f_0' = 1/A \{ \alpha \cos \sqrt{\alpha} - \sqrt{\alpha}/z \cos \sqrt{\alpha} z/2 \} e^{nt}$$

$$f_0'' = 1/A \left[\frac{\sqrt{\alpha}}{4z^{3/2}} \cos \sqrt{\alpha} z + 1/4 \left(\alpha/z \right) \sin \alpha z \right] e^{nt}$$

$$\begin{aligned} f_1' = & [G/2\sqrt{\beta/z} \cos \sqrt{\beta z} - 2p_1/\beta - \alpha/2A^2 \beta (2\cos^2 \alpha + 1) + \cos \sqrt{\alpha} z/4\sqrt{z(\beta-4\alpha)} - \cos \\ & \sqrt{\alpha} \cos \sqrt{\alpha} z/(\beta-\alpha)\sqrt{z}] e^{2nt} \end{aligned}$$

$$f1// = [-G\sqrt{\beta} / 4z^{3/2} \cos\sqrt{\beta}z - G\beta/2z \sin\sqrt{\beta}z - \cos 2\sqrt{\alpha}z / 8Z^{3/2}(\beta - 4\alpha) - \sqrt{\alpha}/2z \sin\sqrt{\alpha}z / (\beta - 4\alpha) + \cos\sqrt{\alpha}\cos\sqrt{\alpha}z/z^{3/2}(\beta - \alpha) + \sqrt{\alpha}\cos\sqrt{\alpha}\sin\sqrt{\alpha}z/z(\beta - \alpha)]e^{2nt}$$

8.0. Computer programming for graphical REPRESENTATION:

P^* is defined as follows

$$P(r,t) - P(R,t) / Re = [K_0(t)\log(r) + Re^* K_1(t) + \dots] \text{-----(42)}$$

$$P^* = [K_0(t)\log(r) + Re^* K_1(t) + \dots] \text{-----(43)}$$

Where $K_0 = P_0 e^{nt}$ and $K_1 = P_1 e^{nt}$

It is assume that $\{P(R, t)\}$ is a Known pressure at some cross-section in the flow domain at $r=R$

A computer programming is done to obtain graphs to show variations of P^* with different Re^* , M & n . The software (Mathematica 4) is used to draw various graphs to show variations of P^* with Re^* , M & n .

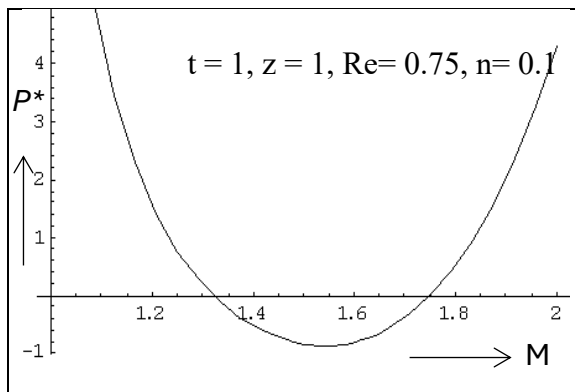


Fig.1: Variation of pressure at the upper surface with magnetic Hartmann number for $n=0.1$

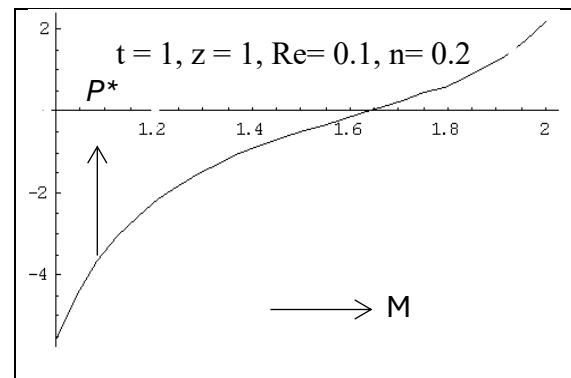


Fig.2: Variation of pressure near central line with magnetic Hartmann number for $n=0.2$

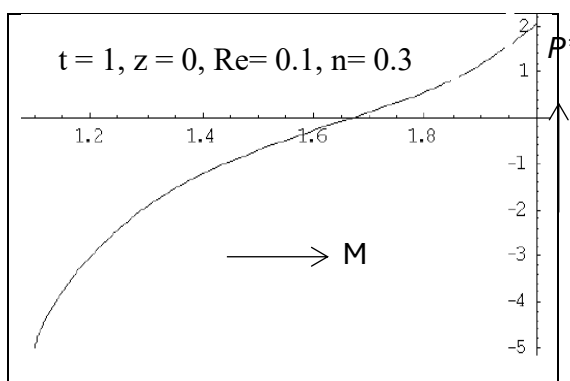


Fig.3: Variation of pressure near central line with magnetic Hartmann number for $n=0.3$

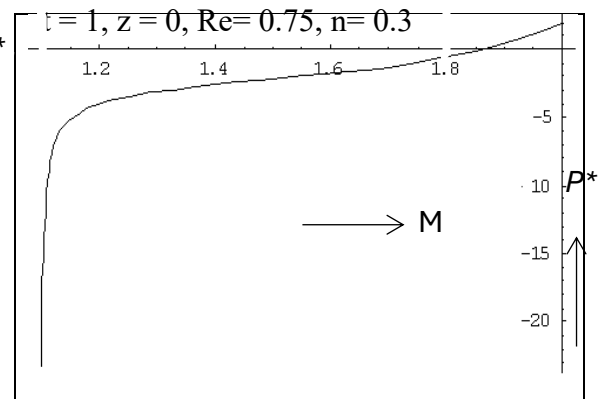


fig.4: Variation of pressure near central line with magnetic Hartmann number for $n=0.75$ and $Re=0.75$

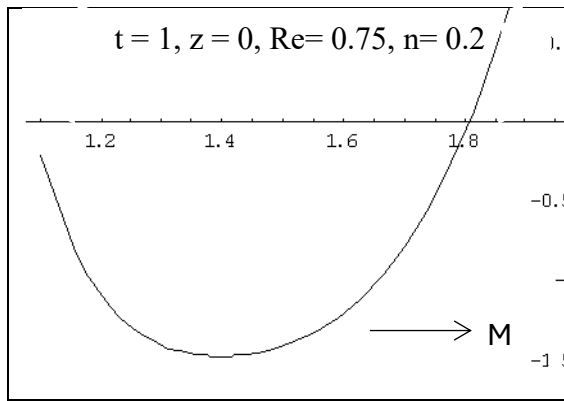


fig.5: Variation of pressure near central line with magnetic Hartmann number for $n=0.2$ and $Re=0.75$

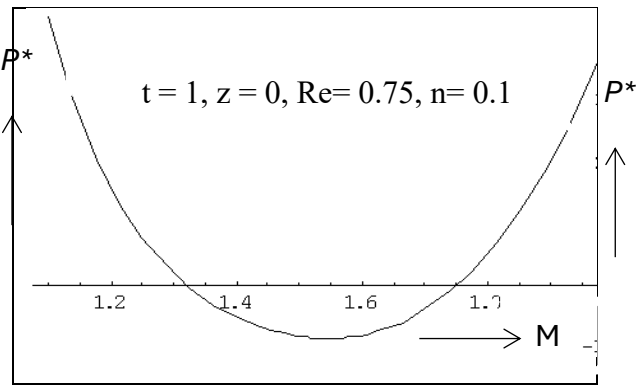


fig. 6: Variation of pressure near central line with magnetic Hartmann number for $n=0.1$ and $Re=0.75$

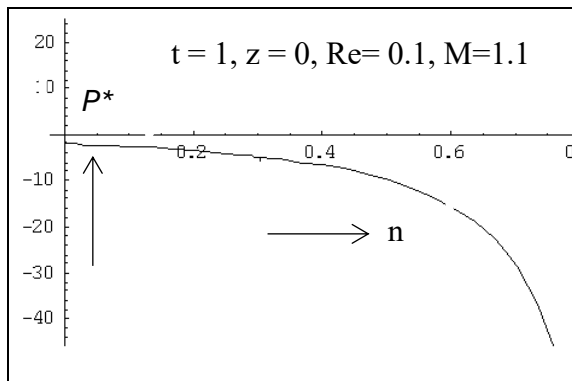


Fig.7: Variation of pressure near central line with growing factor for $M=1.1$

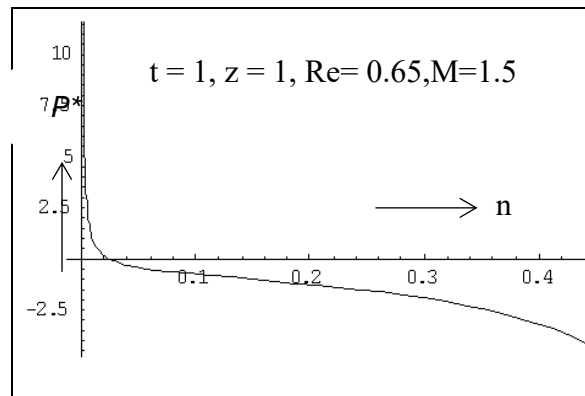


fig. 8 Variation of pressure near central line with growing factor for $Re=0.65$ and $M=1.5$

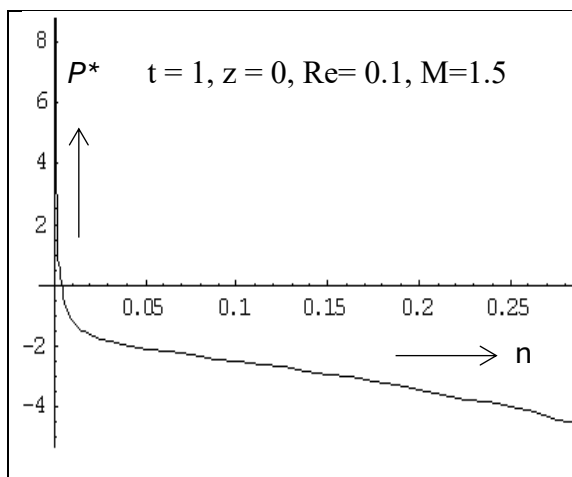


Fig. 9: Variation of pressure near central line with growing factor for $Re=0.1$ and $M=1.5$

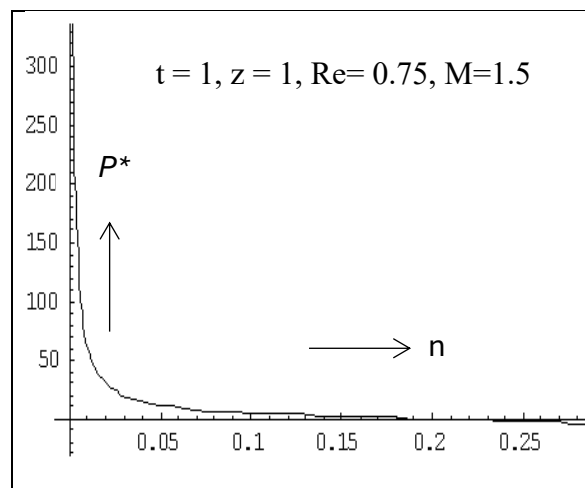


fig. 10: Variation of pressure near central line with growing factor for $Re=0.75$ and $M=1.5$

9.0 Results and discussion analysing GRAPHS:

We have calculated P^* for the various values of reduced Reynolds number Re^* . Their variations with magnetic field parameter (M) and growing factor (n) are shown graphically in figures (Fig.1-10). The results obtained are some cases significantly different from results obtained due to exponentially decay source. As the perturbation technique is used for small values of Re^* therefore, results are valid for at a large distance from the source from Z -axis while $M=0$ in the ordinate axis means in the absence of the magnetic field. [10][11] [12]

The variation of pressure with an increasing magnetic field depends on the growing factor for a growing source, similar to how it depends on the decay factor for a decaying source. The effects of a reduced Reynolds number on pressure under a uniform magnetic field are more pronounced for a growing source than for a decaying source, particularly at smaller values of the growth factor (n). For a growing source, at lower magnetic field values, pressure initially increases slowly but then rises rapidly as the growth factor increases. In contrast, for a decaying source, pressure first increases and then decreases as the decay factor (n) increases. The influence of a reduced Reynolds number on pressure is more significant for a growing source but negligible for an exponentially decaying source. Additionally, non-linear convective inertia effects on pressure become significant for all values of the growth factor in a growing source, while they only become significant for small decay factors in an exponentially decaying source. [16][17] [18]

Pressure (P^*):

Effect of Re^* : The effect of Re^* on P^* for all values of M and n are significant, figs. (1-3). But for decay source the effect of Re^* on P^* for all values of M and n is negligibly small.

Effect of M : P^* first decreases slowly and then increase steadily with the increase of M at constant n , figs (1-3). The value of M from where P^* increases also increases with the increase of n , fig (4,5), the nature is same for decay source.

Effect on n : At lower value of M , P^* with the increase of n , first increases slowly and then increases rapidly for constant M , figs (5.6). The effects of Reynolds number on P^* is significant for high value of M (fig. 7-10). But for a decay source, P^* with the increase of n , first increases then decreases with the increase of n . [12][14] [15] [19][20]

10.0 CONCLUSIONS:

In this theoretical study of pressure distribution in magnetohydrodynamics (MHD), we have examined the influence of various factors, such as the magnetic field, Reynolds number, and source behavior (growing vs. decaying), on the pressure profile. The results highlight that the pressure variation in the presence of a uniform magnetic field is significantly influenced by whether the source is growing or decaying. For a growing source, pressure increases slowly at first with the magnetic field, followed by a rapid rise as the growth factor increases. In contrast, for a decaying source, the pressure initially increases but then decreases with a higher decay factor. The effects of a reduced Reynolds number on the pressure are more pronounced in the case of a growing source, whereas they have little impact on a decaying source. Moreover, the study demonstrates that non-linear convective inertia effects are crucial in determining pressure

distribution for a growing source across all values of the growth factor. For decaying sources, however, these effects only become significant at smaller decay factors.

This study provides valuable insights into the complex interactions between magnetic fields, Reynolds number, and source behavior in MHD flows, which can be crucial for applications involving magnetically controlled fluids and plasma dynamics. Further experimental and computational studies are needed to validate these theoretical predictions and explore more intricate MHD phenomena.

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